DMAP for Base Excitation Modal Participation Factors and Effective Mass

1. Development of Modal Participation Factor and Effective Mass Equations

A common structural analysis procedure is to analyze a structure to predict responses that will occur during a sinusoidal or random vibration test in which a predetermined motion is imparted by a shaker at the structure/shaker interface. The response analyses are often performed in two stages: a modal analysis to determine the eigenvalues and eigenvectors “cantilevered” at the shaker interface, followed by a steady state frequency response analysis using the modal information. The output of the modal analysis, while giving information that is useful in comparing eigenvalues and eigenvectors between the analysis and test, does not give a sufficiently clear indication of which modes will be important contributors in the subsequent frequency response analysis.

By using the eigenvector data obtained in the normal mode analysis, modal participation factors (MPF’s) and effective masses (EM’s) can be calculated which do give a clear indication of the relative importance of each mode in terms of its response to any base motion input. These factors, like the generalized mass or stiffness, (automatically calculated in NASTRAN and printed in the eigenvalue summary), are a property of the structure. In addition, however, they are also a property of the form of the base acceleration.

The procedure presented herein provides a technique for calculating the MPF’s and EM’s for base excitation problems using a DMAP alter to the NASTRAN real eigenvalue analysis Rigid Format.

To derive an equation for the modal participation factors due to base excitation, consider a general structural model as shown in the figure below:

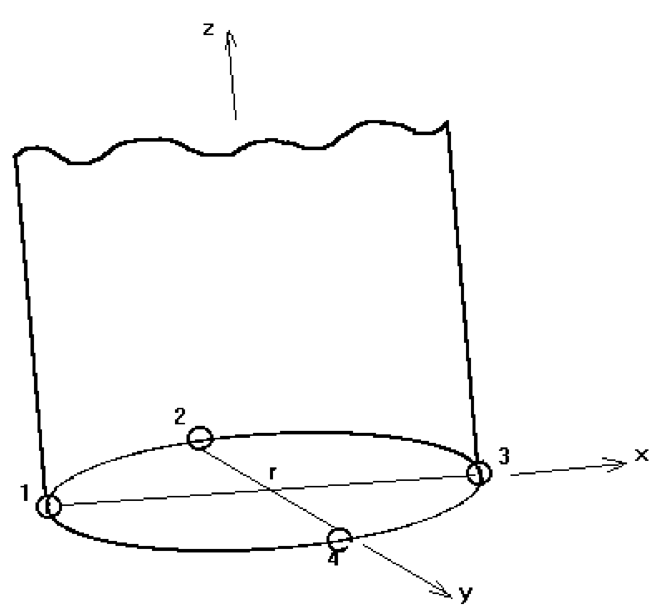


Figure 1

Presumably, this structure would be bolted to a vibration table (shaker) at several points, denoted in this example as points 1 through 4. This situation can be modeled in NASTRAN by creating some central point, r, and connecting 1-4 to r with a rigid element. Then r represents a set of nonredundant degrees of freedom (DOF) at which the shaker motion can be specified. The equations of motion of this structure, using the displacement set notations of NASTRAN, and ignoring damping for the moment, are:

$$M_{gg}\ddot{u}_g + K_{gg}u_g = P_g \qquad 1.$$

where P_g contains the loads necessary to drive the r DOF by the specified base motions imparted by the shaker.

Following the elimination of multi-point constraints, single point constraints, and DOF omitted via Guyan reduction (using ASET or OMIT), the g set equations of motion are reduced to the a set equations of motion:

$$M_{aa}\ddot{u}_a + K_{aa}u_a = P_a \qquad 2.$$

where

$$u_a = \begin{Bmatrix} u_l \\ u_r \end{Bmatrix} \qquad 3.$$

In the reference [1] to the original DMAP written for calculation of MPF's, use was made of the SUPORT feature in NASTRAN . The r-set (on Bulk data SUPORT cards) was used as the shaker interface. The equations developed, below, for the MPF's and EM's use this concept but the implementation described in this DMAP does not use SUPORT DOF's. There is a limitation to the DMAP, that was not in the original, which will be addressed in Section 1.1 later.

As mentioned, the r DOF would ordinarily be identified on SUPORT bulk data cards in the NASTRAN data deck. The l DOF are all remaining DOF in the a set. Partitioning equation 2 into the r and l sets:

$$\begin{bmatrix} M_{ll} & M_{lr} \\ M_{lr}^T & M_{rr} \end{bmatrix}\begin{Bmatrix} \ddot{u}_l \\ \ddot{u}_r \end{Bmatrix} + \begin{bmatrix} K_{ll} & K_{lr} \\ K_{lr}^T & K_{rr} \end{bmatrix}\begin{Bmatrix} u_l \\ u_r \end{Bmatrix} = \begin{Bmatrix} 0 \\ P_r \end{Bmatrix} \qquad 4.$$

where P_r are the forces exerted by the shaker on the structure and result in the base motions u_r. The forces P_r are not known at this point; rather the motions at the r DOF are known. In this context, the forces in P_r are really forces of constraint which are necessary to produce the desired base motion, u_r. Notice in equation 4 that there are no applied forces on the l DOF. The situation being simulated here is one in which the only excitation comes about due to the shaker motion driving the structure at the r DOF.

The motions at the l DOF, u_l, can be considered to be made up of contributions due to rigid body motion from the r DOF plus elastic motion relative to the r DOF. That is, we can write:

$$u_l = u_l^r + u_l^e \qquad 5.$$

[1] W. Case, "*A NASTRAN DMAP Procedure For Calculation of Base Excitation Modal Participation Factors*", Eleventh NASTRAN Users' Colloquium, May 5-6 1983, in San Francisco, CA

where u_l^r are the rigid body displacements of the l DOF due to the (shaker) r DOF and u_l^e are the elastic displacements of the l DOF relative to the r DOF. Thus u_l^e would be the displacements for a structure that was constrained in the r DOF. The rigid body displacements, u_l^r, of the l DOF due to the r DOF can be determined from geometric considerations and can be written as:

$$u_l^r = R_{lr} u_r \tag{6.}$$

Matrix R_{lr} can be generated using DMAP modules VECPLOT and PARTN. The columns of R_{lr} are rigid body displacements of the l DOF due to unit motion at one of the r DOF (since the r DOF are a nonredundant set of DOF). Substituting equations 5 and 6 into equation 4 the two matrix equations for the unknowns u_l^e and P_r are:

$$M_{ll}\ddot{u}_l^e + K_{ll}u_l^e = -(M_{ll}R_{lr} + M_{lr})\ddot{u}_r \tag{7.}$$

$$P_r = M_{lr}^T\ddot{u}_l^e + K_{lr}^T u_l^e + (M_{lr}^T R_{lr} + M_{rr})\ddot{u}_r \tag{8.}$$

where use of the fact that (since the r DOF are a kinematic set):

$$\begin{gathered} K_{rr} = K_{lr}^T K_{ll}^{-1} K_{lr} \\ R_{lr} = -K_{ll}^{-1} K_{lr} \\ \therefore K_{ll}R_{lr} + K_{lr} = 0 \end{gathered} \tag{9.}$$

has been made (as discussed in the NASTRAN Theoretical Manual). Equation 7 is the well known form for base acceleration input where the problem is solved for motions relative to the base using an excitation that is the equivalent inertia loading on the l DOF. Equation 8 solves for the forces required to drive the base at the specified acceleration, $\ddot{u}_r$. A better form for P_r results if equations 9 and 7 are used to eliminate the stiffness term in equation 8. The result is:

$$P_r = (M_{ll}R_{lr} + M_{lr})^T \ddot{u}_l^e + \overline{M}_{rr}\ddot{u}_r \tag{10.}$$

where

$$\overline{M}_{rr} = R_{lr}^T M_{ll} R_{lr} + R_{lr}^T M_{lr} + M_{lr}^T R_{lr} + M_{rr} \tag{11.}$$

is the rigid body mass matrix of the structure relative to the r DOF. Notice in equation 10 that if the elastic deformations , u_l^e , are zero then P_r is equal to the rigid body mass matrix times the rigid body accelerations (as it should be).

Equations 7 and 10 are the equations to solve for the motions, u_l^e, of the l DOF relative to the r DOF and for the forces, P_r, required to produce the specified accelerations, $\ddot{u}_r$, at the r DOF. In order to develop equations for the modal participation factors (MPF's) and effective masses (EM's) it is convenient to transform equations 7 and 10 to modal DOF. To that end, define f_{l_j} to be the j-th eigenvector of the structure when constrained at the r DOF and define Φ_{ln} to be a matrix whose columns are the eigenvectors f_{l_j}. Also, define a set of modal DOF, $x_j\,(j = 1,2,\ldots,n \le l)$ and a modal DOF vector

$$Z_n = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix}$$

Then the modal transformation is:

$$u_l^e = \Phi_{\ln} Z_n = \begin{bmatrix} f_{l_1} & f_{l_2} & \cdots & f_{l_n} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix} = \sum_{j=1}^{n} f_{l_j} x_j \qquad 12.$$

Substituting equation 12 into equation 7 and premultiplying by $\Phi_{\ln}^T$ the result is:

$$\hat{M}_{nn} \ddot{Z}_n + \hat{K}_{nn} Z_n = -\Phi_{\ln}^T (M_{ll} R_{lr} + M_{lr}) \ddot{u}_r \qquad 13.$$

where

$$\hat{M}_{nn} = \Phi_{\ln}^T M_{ll} \Phi_{\ln} = \begin{bmatrix} m_1 & & & \\ & m_2 & & \\ & & \ddots & \\ & & & m_n \end{bmatrix} \qquad 14.$$

and

$$\hat{K}_{nn} = \Phi_{\ln}^T K_{ll} \Phi_{\ln} = \begin{bmatrix} k_1 & & & \\ & k_2 & & \\ & & \ddots & \\ & & & k_n \end{bmatrix} \qquad 15.$$

are both diagonal matrices of m_j generalized masses and k_j generalized stiffnesses for the j-th mode. Thus, equations 13 can be written in individual form (for each mode) as:

$$m_j \ddot{x}_j + k_j x_j = -f_{l_j}^T (M_{ll} R_{lr} + M_{lr}) \ddot{u}_r \qquad j = 1,2,\ldots,n \le l$$

At this point, modal damping can be added. With damping, and with dividing by the generalized mass, m_j, the above equation can be written as:

$$\begin{gathered} \ddot{x}_j + 2 g_j w_j \dot{x}_j + w_j^2 x_j = -f_{r_j} \ddot{u}_r \\ w_j^2 = k_j / m_j \end{gathered} \qquad 16.$$

where g_j is the j-th mode structural damping (entered in NASTRAN via a TABDMP1 bulk data card), w_j is the j-th mode resonant frequency and

$$f_{r_j} = \frac{1}{m_j} f_{l_j}^{T} (M_{ll} R_{lr} + M_{lr}) \qquad 17.$$

The row matrix f_{r_j} contains as many terms as there are DOF identified on SUPORT bulk data cards as being base acceleration DOF (or shaker DOF). The solution to equation 16 for any one component of base acceleration is seen, from the form of equation 16, to be proportional to the corresponding term in f_{r_j}. Thus, the f_{r_j} are known as modal participation factors (MPF). A list of all MPF's for the modes will therefore identify, through the relative magnitudes of the f_{r_j}, the modes which will contribute most significantly to the solution to a base acceleration (shaker excitation) problem.

As an example of this, consider the situation when the base acceleration is a steady state sinusoidal input at the resonant frequency w_j. In this case:

$$\ddot{u}_r = \bar{\ddot{u}}_r \sin w_j t \qquad 18.$$

where $\bar{\ddot{u}}_r$ is the magnitude of the base acceleration in the r DOF. The solution to equation 16 for this excitation is:

$$\ddot{x}_j = \bar{\ddot{x}}_j \sin(w_j t - p/2) \qquad 19.$$

where $\bar{\ddot{x}}_j$ is the modal acceleration amplitude:

$$\begin{aligned} \bar{\ddot{x}}_j &= f_{r_j} Q_j \bar{\ddot{u}}_r \\ Q_j &= 1/2g_j \end{aligned} \qquad 20.$$

and Q_j is the modal amplification factor. The significance of the MPF is evident from equation 20. At resonance, the modal acceleration amplitude, $\bar{\ddot{x}}_j$, is equal to the product of the MPF, the modal amplification factor and the base acceleration. For systems with relatively small damping, the magnitude of the accelerations at grid points throughout the structure can be approximated by a single mode response (the resonating mode) so that, using only one mode in equation 12:

$$\bar{\ddot{u}}_l^{e} \approx f_{l_j} \bar{\ddot{x}}_j = f_{l_j} f_{r_j} Q_j \bar{\ddot{u}}_r \qquad 21.$$

Equation 21 represents a single mode estimate to the *elastic* portion of the grid point accelerations and, as seen from equation 18, 19 and 21, the elastic portion *lags* the input acceleration by 90 degrees. The total acceleration is calculated from equations 5 and 6 and includes, besides the elastic portion, u_{l_e}, the rigid portion in equation 6 (which is *in phase* with the input acceleration).

The MPF, together with the eigenvectors calculated in a real eigenvalue analysis can therefore be used to obtain estimates of the magnitudes of grid point accelerations (that is, estimates of those that would be found in a subsequent forced base frequency response analysis). These MPF can be easily calculated during the real eigenvalue analysis run. A matrix of the MPF for all modes (equation 17 are the ones for the j-th mode) can be written as:

$$F_{r_n} = \left\{ f_{r_j} \right\} = \hat{M}_{nn}^{-1} \Phi_{\ln}^{T} (M_{ll} R_{lr} + M_{lr}) \qquad 22.$$

That is, each row of F_{r_n} is an f_{r_j} . Each *column* of F_{r_n} gives the MPF for all modes for one DOF of base acceleration. For example, column 1 of F_{r_n} contains the MPF for base acceleration in the first DOF in the r set.

The MPF can also be used to easily calculate what is commonly referred to as effective masses (EM's). Consider equation 10 for the base loads (shaker forces required to produce $\ddot{u}_r$). Using the modal transformation in equation 12, the loads P_r in equation 10 can be written as:

$$P_r = \sum_{j=1}^{n} P_{r_j} + \overline{M}_{rr} \ddot{u}_r \qquad 23.$$

where

$$P_{r_j} = (M_{ll} R_{lr} + M_{lr})^T f_{l_j} \ddot{x}_j$$

Using equation 20, the amplitude of the elastic portion of the shaker forces are;

$$\overline{P}_{r_j} = (M_{ll} R_{lr} + M_{lr})^T f_{l_j} f_{r_j} Q_j \overline{\ddot{u}}_r$$

Using equation 17 for the f_{r_j} :

$$\overline{P}_{r_j} = (M_{ll} R_{lr} + M_{lr})^T f_{l_j} \frac{1}{m_j} f_{l_j}^T (M_{ll} R_{lr} + M_{lr}) Q_j \overline{\ddot{u}}_r$$

or

$$\overline{P}_{r_j} = \tilde{M}_{rr_j} Q_j \overline{\ddot{u}}_r \qquad 24.$$

where:

$$\tilde{M}_{rr_j} = (M_{ll} R_{lr} + M_{lr})^T f_{l_j} \frac{1}{m_j} f_{l_j}^T (M_{ll} R_{lr} + M_{lr}) \qquad 25.$$

is the j-th mode effective mass (EM). It is a square matrix of size r, and from equations 23 and 24 it is seen that its use is in determining the j-th mode contribution to the base forces (required shaker forces). For systems with small damping, the shaker force magnitudes in the j-th mode at resonance would be approximately those given by equation 24. It is interesting to note that if the sum of all effective mass matrices is denoted as:

$$\tilde{M}_{rr} \equiv \sum_{j=1}^{n} \tilde{M}_{rr_j} \qquad 26.$$

and if <u>all</u> n eigenvectors are used then:

$$\tilde{M}_{rr} = (M_{ll} R_{lr} + M_{lr})^T M_{ll}^{-1} (M_{ll} R_{lr} + M_{lr}) \qquad 27.$$

If there are no masses at the r DOF (the "base" of the structure) then equation 27 is:

$$\tilde{M}_{rr} = \overline{M}_{rr}$$

That is, the sum of all effective mass matrices is the rigid body mass matrix when all masses are at the l DOF. Thus, effective masses can be calculated in a real eigenvalue analysis and can be used to estimate how many modes would have to be used in a subsequent dynamic analysis to make sure that a high percentage of the complete system mass is included in the response. This could be especially useful in transient analyses in which a single mode response at resonance may not be a valid approximation. The effective mass matrices in equation 25 can be determined from the MPF. As seen from equations 17 and 25:

$$\tilde{M}_{rr_j} = m_j f_{r_j}^T f_{r_j} \qquad 28.$$

That is, the j-th mode effective mass matrix is an r x r matrix equal to the product of the MPF and its transpose and the j-th mode *generalized* mass.

1.1 Implementation Into DMAP Without Using The NASTRAN "SUPORT" Feature

The implementation into DMAP described in the reference used the SUPORT feature in NASTRAN which is somewhat of an inconvenience in that users generally do not use this feature in their fixed base normal modes analyses. In order to avoid this inconvenience, the current DMAP uses the s-set (single point contrained DOF's) to describe the shaker interface. The only disadvantage is that the M_{lr} terms in equation 22 cannot be implemented since there may be other DOF's in the s-set than those associated with the shaker interface. Thus, the DMAP described herein ignores the M_{lr} terms. These terms are mass <u>couplings</u> between the shaker DOF's and the free (l set) DOF's which in many instances are probably negligible. However, the DMAP described in the reference does not make this assumption, and, as such, is a safer choice for the DMAP implementation.

DMAP Alter for Generation of Modal Participation Factors and Effective Mass

The DMAP alters for calculating the MPF's and EM's are given below. The calculations are done in MSC NASTRAN Solution 103 (real eigenvalue analysis).

DMAP Alter

```
$
$ MSC v 70.5 Modal Participation factor (MPF) and Effective Mass (EM) DMAP

$ This DMAP will calculate MPF's and EM's for a model of a structure that is
$ constrained (to zero motion) at a set of DOF's that would represent an
$ interface to a vibration shaker. The MPF's and EM's are for base motion in
$ 6 DOf's at this "shaker" interface. The matrices calculated and printed out
$ are (note that NEIGV is the number of eigenvectors computed in subdmap
$ MODERS and that the 6 degrees of shaker motion are in the global coordinate
$ system at grid point MPFPNT, which is defined later):
$
$     1) PARFAC    : Matrix of MPF's for NEIGV modes and 6 degrees of shaker
$                    motion. There are 6 rows with each row giving the MPF's
$                    for modes 1 through NEIGV. Printing of PARFAC is skipped
$                    unless the user inputs parameter PPARFAC = 1 in Bulk Data.

$     2) EFFMASSO  : Matrix of effective masses (or weights) for NEIGV modes
$                    and 6 degrees of shaker motion. There are 6 rows with
$                    each row giving the EM's for modes 1 through NEIGV.
$                    The units of EFFMASSO are the same as the units in which
$                    the Grid Point Weight Generator output is printed, and
$                    this depends on what the user inputs as mass units and
$                    PARAM WTMASS in Bulk Data.

$     3) MASSSUM   : The sum of each row in EFFMASSO gives the total mass
$                    (or weight) in all modes for each of the 6 DOF's of
$                    shaker motion. This is a 6 x 1 matrix.

$     4) PMASSSUM  : The ratio of each of the terms in MASSSUM to the rigid
$                    body mass (or weight) expressed as a percent. The rigid
$                    body mass (weight) is taken from the diagonal of the
$                    6 x 6 rigid body mass (weight) matrix calculated relative
$                    to grid point MPFPNT (defined below). This is a 6 x 1
$                    matrix.

$ Parameters Used (default values in parentheses can be overriden on Bulk
$ Data PARAM cards):

$     1) MPFPNT    : Grid point about which to calculate rigid body mass
$                    and displacement properties to use in generating the
$                    MPF's and EM's. If this is input, then it MUST be
$                    one of the points constrained in 6 DOF on SPC Bulk
$                    Data cards. The default value is 0, the basic coord
$                    system origin.

$     2) PPARFAC   : Print matrix PARFAC if = 1 (default = 0, no print)

$ IT IS IMPORTANT TO REMEMBER THAT THE MPF'S AND EM'S ARE CALCULATED FOR A
$ STRUCTURE WITH A BASE MOTION THAT IS IN 6 DOF'S AT GRID POINT MPFPNT WITH
$ ALL OTHER SPC'D DOF'S (EXCEPT THOSE SPC'D BECAUSE OF SINGULARITIES) MOVING
$ THE SAME AS THAT AT GRID POINT MPFPNT.
```

```
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
COMPILE SUBDMAP=SEMODES,SOUIN=MSCSOU,NOREF,LIST

 1 ALTER 51

$ Generate rigid body modes in global coordinates

 2   VECPLOT, ,BGPDTS,EQEXINS,CSTMS,,/RBGLT/V,Y,MPFPNT=0//4                $
 3   TRNSP     RBGLT/RBGL                                                  $

$ Calculate G set RB mass relative to MPFPNT

 4   SMPYAD    RBGL,MGG,RBGL,,,/MR6/3////1                                 $

$ Store MAA, RBGL and MR6 on database

 5   CALL      DBSTORE MAA,RBGL,MR6,,//1/1/'     '/0/                      $

 6 ENDALTER

COMPILE SUBDMAP=MODERS,SOUIN=MSCSOU,NOREF,LIST

 7 ALTER 108

 8   TYPE      PARM,,RS,Y,WTMASS                                           $
 9   TYPE      PARM,,RS,N,ACCGRAV                                          $
10   TYPE      PARM,,CS,N,COMGRAV                                          $

$ Fetch A set mass matrix and RBGL from the data base

11   CALL      DBFETCH /MAA,RBGL,MR6,,/1/1/0/0/0                           $

$ Generate partitioning vector for G to A set partition

12   VEC       USET/VGRC/'G'/'A'/'COMP'                                    $

$ Partition rigid body modes to A-set size

13   PARTN     RBGL,,VGRC/RBAA,,,/1                                        $

$ Invert generalized mass matrix

14   DIAGONAL  MI/IMI/'SQUARE'/-1.0                                        $

$ Form the product [M]*[PHI]

15   MPYAD     MAA,PHA,/MPHI                                               $

$ Form the product [M*PHI]*IMI

16   MPYAD     MPHI,IMI,/MMPHI/0                                           $

$ Sum rigid body component contributions

17   MPYAD     RBAA,MMPHI,/PARFAC/1                                        $
```

```
$ Print modal participation factors if parameter PPARFAC says to

18   PARAM    //'NOP'/V,Y,PPARFAC=0                                        $
19   IF (PPARFAC = 1) THEN                                                 $
20     MESSAGE  //'**************************************************'/
                  '**************************************************'/    $
21     MESSAGE  //'PARFAC is a matrix of modal participation factors'/
                  'for base motion at the MPFPNT'/
                  '(specified by the user)'/                               $
22     MESSAGE  //'**************************************************'/
                  '**************************************************'/    $
23     MATPRT PARFAC//                                                     $
24   ENDIF

$ Square each term in matrix to form PF(**2)

25   DIAGONAL  PARFAC/MPFSQ/'WHOLE'/2.                                     $

$ Form effective mass

26   MPYAD     MPFSQ,MI,/EFFMASS/0                                         $

$ Scale mass by gravity constant (1.0/WTMASS value)
$ Convert to complex form.

27   ACCGRAV = 1.0/WTMASS                                                  $
28   COMGRAV = CMPLX(ACCGRAV)                                              $

$ Multiply the effective mass by COMGRAV to get effective mass in the
$ same units as output in the Grid Point Weight Generator

29   ADD       EFFMASS,/EFFMASSO/V,N,COMGRAV                               $
30   MESSAGE  //'**************************************************'/
                '**************************************************'/      $
31   MESSAGE   //'EFFMASSO are the effective masses (weights).'/
                 ' Units same as those in Grid Point Weight Generator'/    $

32   MESSAGE  //'**************************************************'/
                '**************************************************'/      $
33   MATPRT    EFFMASSO//                                                  $

$ Generate identity matrix of size NEIGV and convert to a column

34   MATGEN,   /SQID/1/S,N,NEIGV                                           $
35   DIAGONAL  SQID/ID/'COLUMN'/1.                                         $

$ Sum and print effective masses (weights) for all modes

36   MPYAD     EFFMASSO,ID,/MASSSUM/0                                      $
37   MESSAGE  //'**************************************************'/
                '**************************************************'/      $
38   MESSAGE   //'MASSSUM is sum of effective masses in EFFMASSO.'/
                 ' Units same as those in Grid Point Weight Generator'/    $
$
39   MESSAGE  //'**************************************************'/
                '**************************************************'/      $
40   MATPRT    MASSSUM//                                                   $

$ Convert 6x6 rigid body mass matrix to weight units

41   ADD       MR6,/WR6/V,N,COMGRAV                                        $
```

```
$ Strip diagonal from rigid body weight and invert

42   DIAGONAL  WR6/IWR6/'SQUARE'/-1.0                                  $

$ Calculate and print sum of % effective weights

43   MPYAD     IWR6,MASSSUM,/DMASSSUM                                  $
44   ADD       DMASSSUM,/PMASSSUM/(100.,0.)                            $
45   MESSAGE  //'*************************************************'/
               '*************************************************'/    $
46  MESSAGE   //'PMASSSUM is sum of effective masses in EFFMASSO'/
               'as a percent of the total RB mass (weight)'/
               ' This is unitless - numbers are %'/                   $
47   MESSAGE  //'*************************************************'/
               '*************************************************'/    $
48   MATPRT    PMASSSUM                                                $

49 ENDALTER
```

Explanation of DMAP Alter Statements

I Generate rigid body modes and mass

1 Alter in subdmap SEMODES after MAA is first available
2-3 Generate rigid body modes matrix RBGL
4 Generate 6x6 rigid body mass matrix MR6
5 Store matrices on the database to be read in to subdmap MODERS, below
6 End alter in SEMODES

II Calculate MPF's (Modal participation Factors) as the transpose of matrix F_{r_n} in equation 22)

7 Alter in subdmap MODERS (where modes are calculated) near end of that subdmap
8-10 Define parameters used in this DMAP
11 Recall data blocks stored on the database in subdmap SEMODES
12 VGRC is a partitioning vector used in partitioning RBGL to the A-set
13 Partition RBGL to the A-set
14 Invert the generalized mass matrix. Data block IMI is matrix $\hat{M}_{nn}^{-1}$ in equation 22.
15 Data block MPHI is the transpose of $\Phi_{\ln}^{T} M_{ll}$ in equation 22
16 Multiply matrices in steps 14-15
17 Data block PARFAC is the transpose of matrix F_{r_n} in equation 22
18-24 Print the MPF's if parameter PPARFAC says to

III Calculate EM's (Effective Masses) and print their diagonals (diagonals of equation 28)

25 Square terms in PARFAC
26 Generate data block EFFMASS which contains the diagonal terms from all of the $\tilde{M}_{rr_j}$ matrices (j = 1 to number of modes) in equation 28. Column i of EFFMASS contains the diagonal terms of the $\tilde{M}_{rr_j}$ matrix.
27-29 Scale EFFMASS so that it is in the same units as the Grid Point Weight Generator
30-33 Print EM's
34-36 Postmultiply EFFMASS by a column of 1's to sum the effective masses in all modes
37-40 Print sum of EM's as data block MASSSUM
41-48 Divide EM's by the diagonal of the rigid body mass and multiply by 100 to get data block PMASSSUM, which is the percent sums of the EM's
49 End this DMAP alter

2. Example Problem 1 - 10 Cell Beam

2.1 Example Problem 1 Description

Figure 2 shows an example problem using the DMAP alter for a 10 cell beam which is to be given a base motion at grid point 11. The aluminum beam is 100 inches long with a weight of 5000 lb. The data deck and output, including the output related to the DMAP alter, are given in the next section.

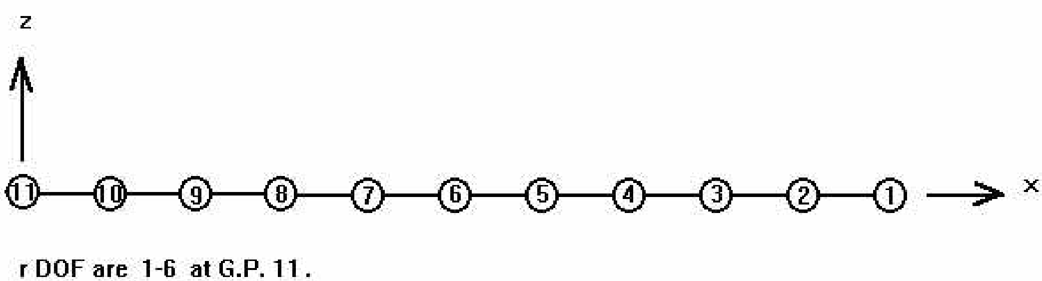


Figure 2 - 10 Cell Beam - Base Excitation at G.P. 11

The first 6 mode shapes are shown in Figure 3. The modal participation factors and the diagonals from the effective mass matrices are shown in Table 1 for the first 21 modes (this covers all of the longitudinal modes, all of the x-z plane lateral modes, and the first torsional mode. The x-y plane lateral eigenvectors are identical to those in the x-z plane; the frequencies are 1000 times greater due to the higher beam moment of inertia in the x-y plane.

Columns 3-6 of Table 1 are the MPF's for all of the DOF's involved in the first 21 modes. Column 3 are those for base motion in DOF 1 at G.P. 11 (i.e. x, or longitudinal, direction), while column 4 is for base motion in the lateral z direction, column 5 for torsion (rotation about x) and column 6 are those for base motion that would be rotation about y at G.P. 11. Investigation of the eigenvector printed output in MPF.PRT reveals that modes 1,3,5,7,11,16-20 are x-z lateral modes, while modes 2,4,6,8-10,12-15 are longitudinal modes and mode 21 is a torsional mode. Columns 3-6 of Table 1 demonstrate this also, in that nonzero values in a column indicate that the mode is in the direction given at the top of the column. The values within a column show the relative importance of that mode in terms of response to a sinusoidal base motion at a modal frequency (as would be calculated from equation 21). In order to show how equation 21 can be used as an approximation to the total response of a structure to sinusoidal base motion, the beam was run in Rigid Format 11 (modal frequency response) with a 1.5 g sinusoidal base acceleration at G.P. 11 in the z direction. Using a modal damping of $g_j = .06667$ (i.e. $Q_j = 15$), the acceleration response at G.P. 1 is shown in Figure 4. Also shown in Figure 4 are the values of G.P. 1 acceleration as calculated using equation 21. As seen in Figure 4, the approximate values are very accurate. The use of a single mode approximation, and using the calculated MPF's, gives a very good estimate of the response over a wide frequency range and can be obtained very easily.

Columns 7-10 of Table 1 give the terms from matrix $\tilde{M}_{rr_j}$, the diagonals from the effective mass matrices. The totals at the bottom of the page give the % of the total rigid body mass (for the appropriate DOF) that the modes above represent. As mentioned earlier, modes 1-21 include all longitudinal (DOF 1) as well as all x-z plane lateral (DOF's 3 and 5) modes. The totals under the DOF 1 and 3 columns only add up to 95% since there is 5% of the total mass at G.P. 11 (the base motion point in the r set). As stated earlier, if there is any mass at the r DOF's the total of the effective masses will not be 100%. The DOF 5 sum is 100% since mass at G.P. 11 has zero moment about G.P.11.

Figure 3 - Modes of 10 Cell Beam

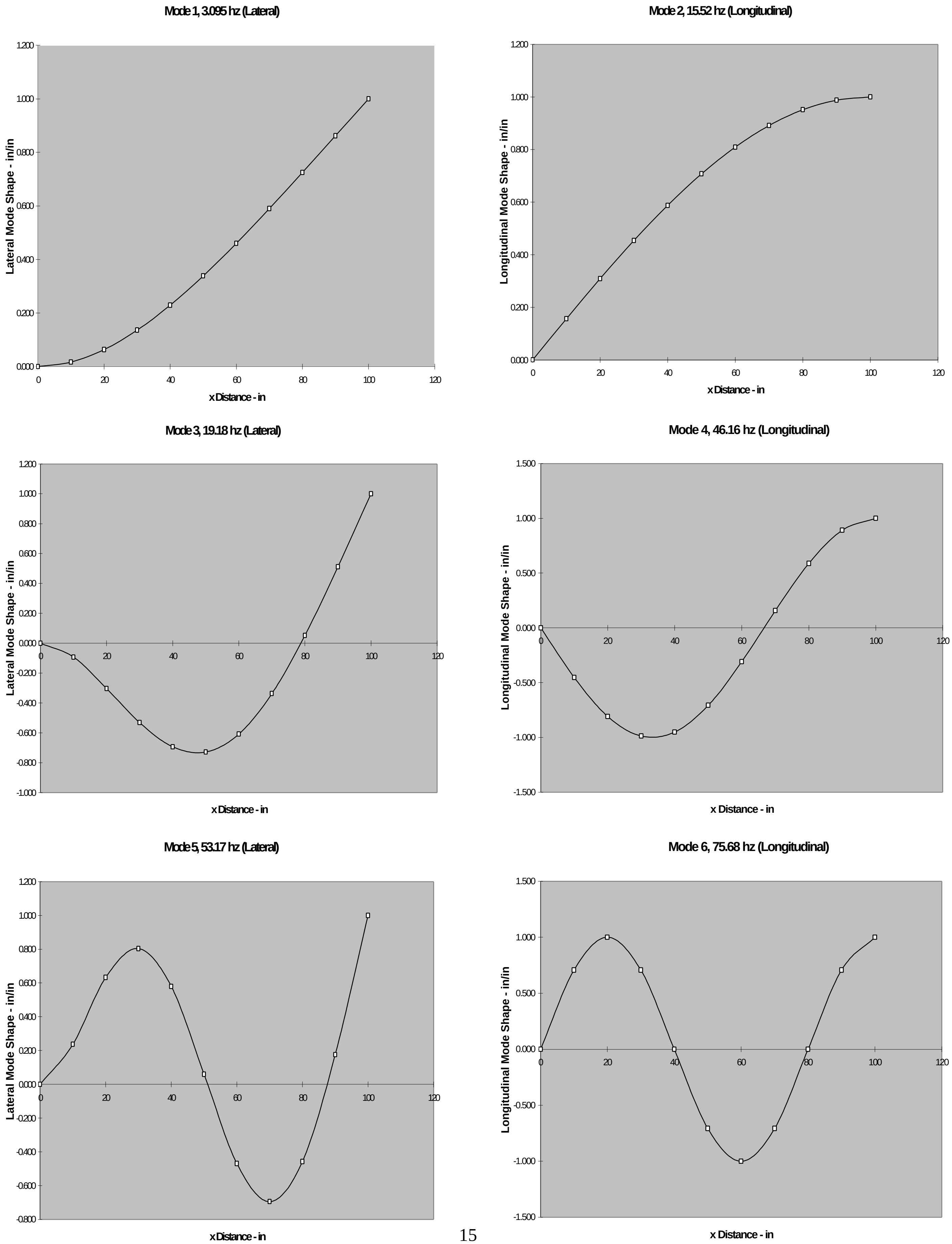
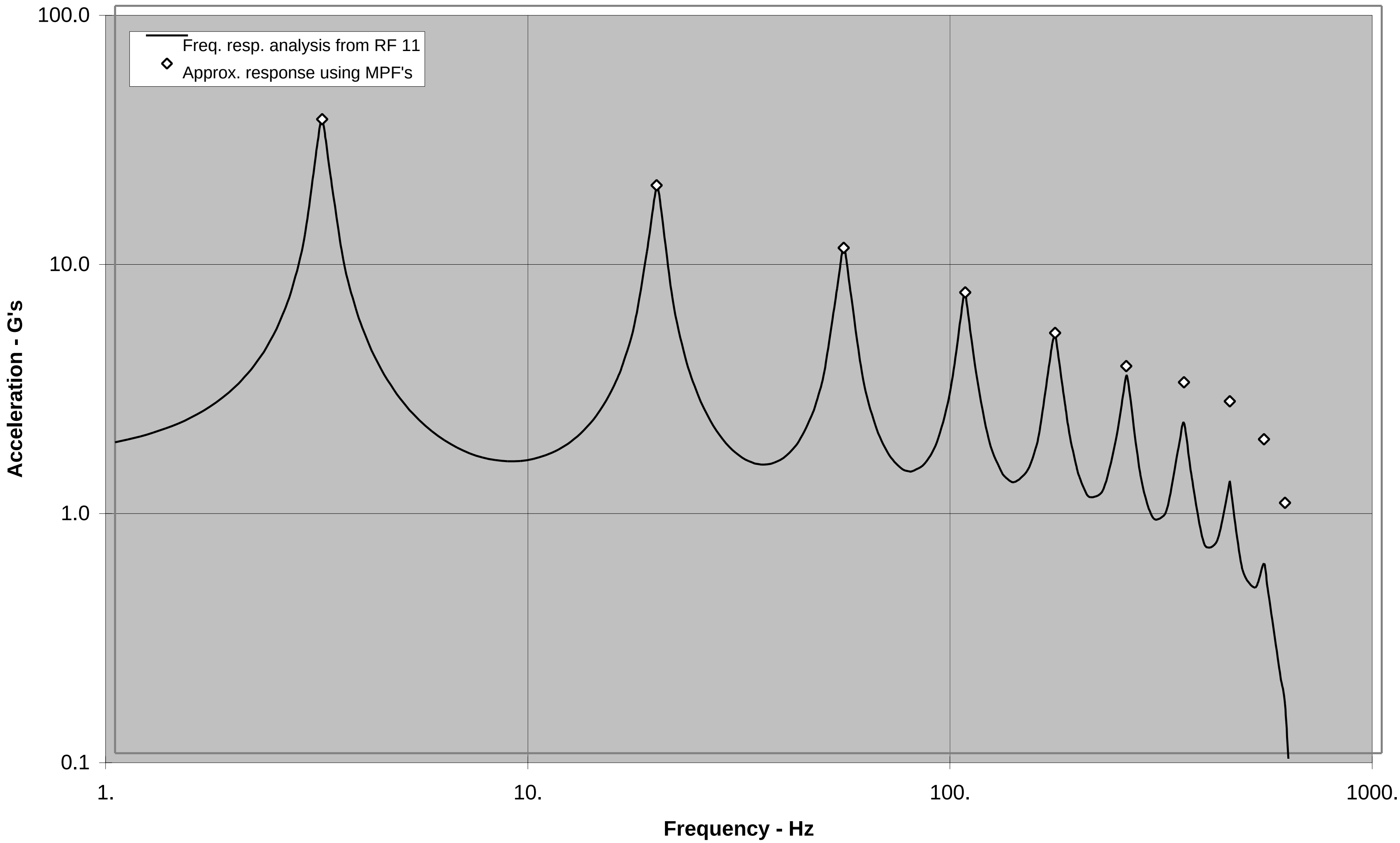


Table 1
Modal Participation Factors (MPF's) and
Effective Masses (EM's) for 10 Cell Beam

Mode No.	Freq. Hz	MPF's				EM's (diagonals, in % of total)			
		DOF 1	DOF 3	DOF 4	DOF 5	DOF 1	DOF 3	DOF 4	DOF 5
1	3.095	.0	1.557	.0	-113.6	.0	61.07	.0	97.03
2	15.51	1.271	.0	.0	0	80.72	.0	.0	.0
3	19.18	.0	-.845	.0	17.80	.0	18.85	.0	2.50
4	16.16	-.417	.0	.0	0	8.68	.0	.0	.0
5	53.17	.0	.474	.0	-6.124	.0	6.47	.0	.32
6	75.68	.241	.0	.0	0	2.91	.0	.0	.0
7	103.1	.0	-.314	.0	2.923	.0	3.30	.0	.09
8	103.3	-.163	.0	.0	0	1.33	.0	.0	.0
9	128.4	.117	.0	.0	0	.69	.0	.0	.0
10	150.4	-.085	.0	.0	0	.36	.0	.0	.0
11	168.6	.0	.216	.0	-1.590	.0	1.99	.0	.03
12	168.6	.061	.0	.0	0	.19	.0	.0	.0
13	182.7	-.041	.0	.0	0	.09	.0	.0	.0
14	192.3	.024	.0	.0	0	.03	.0	.0	.0
15	197.1	-.008	.0	.0	0	.0	.0	.0	.0
16	248.4	.0	-.159	.0	.980	.0	1.31	.0	.01
17	340.0	.0	.137	.0	-.737	.0	.91	.0	.01
18	436.9	.0	.115	.0	-.562	.0	.62	.0	.01
19	526.3	.0	-.081	.0	.367	.0	.36	.0	.00
20	589.9	.0	-.045	.0	.199	.0	.12	.0	.00
21	2592.	.0	.0	1.267	0	.0	.0	77.08	.0
Totals:						95.00	95.00	77.08	100.00

Figure 4
Acceleration at G.P. 1 of 10 Cell Beam
Due to 1.5 g Sinusoidal Base Acceleration at G.P. 11



2.2 Output for Example 1: MPF.PRT

DMAP For Modal Participation Factors and Effective Mass
Example 1 - 10 Cell beam

MPF.PRT

N A S T R A N   E X E C U T I V E   C O N T R O L   D E C K   E C H O

```
ID MPF,EXAMPLE01
APP DISP
SOL 103
TIME 5
DIAG 8
$INCLUDE '/u7/case/Convert_UAI_to_MSC/MPF/MPF_v705.dmp'
CEND
```

C A S E   C O N T R O L   D E C K   E C H O

```
CARD
COUNT
 1      TITLE = DMAP FOR MODAL PART. FACTORS AND EFFEC. MASS FOR BASE MOTION.
 2      SUBTITLE = CANTILEVERED BEAM
 3      ECHO = UNSORT
 4      SPC = 1
 5      METHOD = 1
 6      OUTPUT
 7        VECTOR = ALL
 8        SPCF = ALL
 9      BEGIN BULK
```

I N P U T   B U L K   D A T A   D E C K   E C H O

```
.   1  ..   2  ..   3  ..   4  ..   5  ..   6  ..   7  ..   8  ..   9  ..  10  .
GRID    1               100.    0.      0.
GRID    2                90.    0.      0.
GRID    3                80.    0.      0.
GRID    4                70.    0.      0.
GRID    5                60.    0.      0.
GRID    6                50.    0.      0.
GRID    7                40.    0.      0.
GRID    8                30.    0.      0.
GRID    9                20.    0.      0.
GRID    10               10.    0.      0.
GRID    11                0.    0.      0.
BAROR                                   0.      0.      1.      1
CBAR    1       1       1       2
CBAR    2       1       2       3
CBAR    3       1       3       4
CBAR    4       1       4       5
CBAR    5       1       5       6
CBAR    6       1       6       7
CBAR    7       1       7       8
CBAR    8       1       8       9
CBAR    9       1       9       10
CBAR    10      1       10      11
PBAR    1       1       .5      40.     40.+6   80.     50.
MAT1    1       10.+6           0.3
CONM2   101     1                                                       +CM01
CONM2   102     2                                                       +CM02
CONM2   103     3                                                       +CM03
CONM2   104     4                                                       +CM04
CONM2   105     5                                                       +CM05
CONM2   106     6                                                       +CM06
CONM2   107     7                                                       +CM07
CONM2   108     8                                                       +CM08
CONM2   109     9                                                       +CM09
CONM2   110     10                                                      +CM10
CONM2   111     11                                                      +CM11
```

I N P U T   B U L K   D A T A   D E C K   E C H O

```
      .   1  ..   2  ..   3  ..   4  ..   5  ..   6  ..   7  ..   8  ..   9  ..  10  .
      +CM01   1.
      +CM02   1.
      +CM03   1.
      +CM04   1.
      +CM05   1.
      +CM06   1.
      +CM07   1.
      +CM08   1.
      +CM09   1.
      +CM10   1.
      +CM11   1.
      SPC1    1       123456  11
      ASET1   1234    1       THRU    10
      EIGR    1       GIV                     21      21              1.-4    +E2
      +E2     MAX
      PARAM   GRDPNT  11
      PARAM   WTMASS  .002591
      PARAM   USETPRT  1
      PARAM   USETSEL -1
      PARAM   PPARFAC  1
      ENDDATA
      INPUT BULK DATA CARD COUNT =      56
TOTAL COUNT=        56
```

```
                              O U T P U T   F R O M   G R I D   P O I N T   W E I G H T   G E N E R A T O R
                                                         REFERENCE POINT =       11
                                                                   M O
                      *  5.000000E+03  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00 *
                      *  0.000000E+00  5.000000E+03  0.000000E+00  0.000000E+00  0.000000E+00  2.500000E+05 *
                      *  0.000000E+00  0.000000E+00  5.000000E+03  0.000000E+00 -2.500000E+05  0.000000E+00 *
                      *  0.000000E+00  0.000000E+00  0.000000E+00  1.100000E+01  0.000000E+00  0.000000E+00 *
                      *  0.000000E+00  0.000000E+00 -2.500000E+05  0.000000E+00  1.675000E+07  0.000000E+00 *
                      *  0.000000E+00  2.500000E+05  0.000000E+00  0.000000E+00  0.000000E+00  1.675000E+07 *
                                                                    S
                                           *  1.000000E+00  0.000000E+00  0.000000E+00 *
                                           *  0.000000E+00  1.000000E+00  0.000000E+00 *
                                           *  0.000000E+00  0.000000E+00  1.000000E+00 *
                               DIRECTION
                          MASS AXIS SYSTEM (S)     MASS              X-C.G.        Y-C.G.        Z-C.G.
                                  X            5.000000E+03      0.000000E+00  0.000000E+00  0.000000E+00
                                  Y            5.000000E+03      5.000000E+01  0.000000E+00  0.000000E+00
                                  Z            5.000000E+03      5.000000E+01  0.000000E+00  0.000000E+00
                                                                   I(S)
                                           *  1.100000E+01  0.000000E+00  0.000000E+00 *
                                           *  0.000000E+00  4.250000E+06  0.000000E+00 *
                                           *  0.000000E+00  0.000000E+00  4.250000E+06 *
                                                                   I(Q)
                                           *  1.100000E+01                             *
                                           *                4.250000E+06               *
                                           *                              4.250000E+06 *
                                                                    Q
                                           *  1.000000E+00  0.000000E+00  0.000000E+00 *
                                           *  0.000000E+00  1.000000E+00  0.000000E+00 *
                                           *  0.000000E+00  0.000000E+00  1.000000E+00 *
```

R E A L   E I G E N V A L U E S

MODE NO.	EXTRACTION ORDER	EIGENVALUE	RADIANS	CYCLES	GENERALIZED MASS	GENERALIZED STIFFNESS
1	19	3.782232E+02	1.944796E+01	3.095239E+00	3.263964E+00	1.234507E+03
2	1	9.503404E+03	9.748540E+01	1.551528E+01	6.477500E+00	6.155830E+04
3	20	1.452556E+04	1.205220E+02	1.918167E+01	3.423721E+00	4.973146E+04
4	2	8.413237E+04	2.900558E+02	4.616381E+01	6.477500E+00	5.449674E+05
5	18	1.116134E+05	3.340860E+02	5.317143E+01	3.736068E+00	4.169954E+05
6	3	2.260851E+05	4.754840E+02	7.567564E+01	6.477500E+00	1.464466E+06
7	17	4.197146E+05	6.478539E+02	1.031091E+02	4.346736E+00	1.824389E+06
8	4	4.214662E+05	6.492043E+02	1.033241E+02	6.477500E+00	2.730048E+06
9	5	6.511506E+05	8.069390E+02	1.284283E+02	6.477500E+00	4.217828E+06
10	8	8.926549E+05	9.448042E+02	1.503703E+02	6.477500E+00	5.782172E+06
11	16	1.121570E+06	1.059042E+03	1.685517E+02	5.514039E+00	6.184378E+06
12	10	1.122339E+06	1.059405E+03	1.686096E+02	6.477500E+00	7.269952E+06
13	9	1.317720E+06	1.147920E+03	1.826971E+02	6.477500E+00	8.535534E+06
14	7	1.459673E+06	1.208169E+03	1.922861E+02	6.477500E+00	9.455033E+06
15	6	1.534302E+06	1.238670E+03	1.971404E+02	6.477500E+00	9.938442E+06
16	15	2.435699E+06	1.560673E+03	2.483888E+02	6.716497E+00	1.635937E+07
17	14	4.563269E+06	2.136181E+03	3.399837E+02	6.261703E+00	2.857384E+07
18	13	7.536524E+06	2.745273E+03	4.369237E+02	5.995436E+00	4.518474E+07
19	12	1.093716E+07	3.307138E+03	5.263474E+02	7.147098E+00	7.816896E+07
20	11	1.373947E+07	3.706679E+03	5.899363E+02	7.376435E+00	1.013483E+08
21	31	2.652774E+08	1.628734E+04	2.592210E+03	1.367914E-02	3.628767E+06
22	29	3.782229E+08	1.944795E+04	3.095238E+03	.0	.0
23	32	2.352074E+09	4.849818E+04	7.718726E+03	.0	.0
24	33	6.340246E+09	7.962566E+04	1.267282E+04	.0	.0
25	40	1.187543E+10	1.089744E+05	1.734382E+04	.0	.0
26	30	1.452556E+10	1.205220E+05	1.918168E+04	.0	.0
27	34	1.846579E+10	1.358889E+05	2.162739E+04	.0	.0
28	36	2.552576E+10	1.597678E+05	2.542784E+04	.0	.0
29	39	3.242802E+10	1.800778E+05	2.866027E+04	.0	.0
30	38	3.855927E+10	1.963651E+05	3.125248E+04	.0	.0
31	37	4.337473E+10	2.082660E+05	3.314657E+04	.0	.0
32	35	4.644652E+10	2.155145E+05	3.430020E+04	.0	.0
33	28	1.116134E+11	3.340860E+05	5.317143E+04	.0	.0
34	27	4.197146E+11	6.478539E+05	1.031091E+05	.0	.0
35	26	1.121570E+12	1.059042E+06	1.685518E+05	.0	.0
36	25	2.435699E+12	1.560673E+06	2.483888E+05	.0	.0
37	24	4.563269E+12	2.136181E+06	3.399838E+05	.0	.0
38	23	7.536524E+12	2.745273E+06	4.369238E+05	.0	.0
39	22	1.093716E+13	3.307138E+06	5.263474E+05	.0	.0
40	21	1.373947E+13	3.706679E+06	5.899362E+05	.0	.0

```
                                                      INTERMEDIATE MATRIX ... PARFAC

                                                                COLUMN      1
1         0.000000E+00       0.000000E+00       1.556931E+00       0.000000E+00      -1.135852E+02       0.000000E+00         6

                                                                COLUMN      2
1         1.270620E+00       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00         6

                                                                COLUMN      3
1         0.000000E+00       0.000000E+00      -8.446314E-01       0.000000E+00       1.779980E+01       0.000000E+00         6

                                                                COLUMN      4
1        -4.165300E-01       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00         6

                                                                COLUMN      5
1         0.000000E+00       0.000000E+00       4.736019E-01       0.000000E+00      -6.123850E+00       0.000000E+00         6

                                                                COLUMN      6
1        -2.414214E-01       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00         6

                                                                COLUMN      7
1         0.000000E+00       0.000000E+00      -3.136745E-01       0.000000E+00       2.923105E+00       0.000000E+00         6

                                                                COLUMN      8
1        -1.631852E-01       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00         6

                                                                COLUMN      9
1         1.170850E-01       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00         6

                                                                COLUMN     10
1        -8.540807E-02       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00         6

                                                                COLUMN     11
1         0.000000E+00       0.000000E+00       2.161311E-01       0.000000E+00      -1.590016E+00       0.000000E+00         6

                                                                COLUMN     12
1         6.128008E-02       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00         6

                                                                COLUMN     13
1        -4.142136E-02       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00         6

                                                                COLUMN     14
1         2.400788E-02       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00         6

                                                                COLUMN     15
1        -7.870170E-03       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00         6

                                                                COLUMN     16
1         0.000000E+00       0.000000E+00      -1.592554E-01       0.000000E+00       9.800463E-01       0.000000E+00         6
```

INTERMEDIATE MATRIX ... PARFAC

```
                                                        COLUMN     17
1         0.000000E+00       0.000000E+00       1.371152E-01       0.000000E+00      -7.370272E-01       0.000000E+00        6

                                                        COLUMN     18
1         0.000000E+00       0.000000E+00       1.154235E-01       0.000000E+00      -5.617958E-01       0.000000E+00        6

                                                        COLUMN     19
1         0.000000E+00       0.000000E+00      -8.061019E-02       0.000000E+00       3.668914E-01       0.000000E+00        6

                                                        COLUMN     20
1         0.000000E+00       0.000000E+00      -4.533904E-02       0.000000E+00       1.986909E-01       0.000000E+00        6

                                                        COLUMN     21
1         0.000000E+00       0.000000E+00       0.000000E+00       1.267311E+00       0.000000E+00       0.000000E+00        6
```

```
^^^*************************************************   *************************************************
 ^^^EFFMASSO ARE THE EFFECTIVE MASSES (WEIGHTS). UNITS SAME AS THOSE IN GRID POINT WEIGHT GENERATOR
 ^^^*************************************************   *************************************************
                                                        INTERMEDIATE MATRIX ... EFFMASSO

                                                                  COLUMN      1
      1        0.000000E+00       0.000000E+00       3.053631E+03       0.000000E+00       1.625253E+07       0.000000E+00        6

                                                                  COLUMN      2
      1        4.036191E+03       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00        6

                                                                  COLUMN      3
      1        0.000000E+00       0.000000E+00       9.426825E+02       0.000000E+00       4.186596E+05       0.000000E+00        6

                                                                  COLUMN      4
      1        4.337431E+02       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00        6

                                                                  COLUMN      5
      1        0.000000E+00       0.000000E+00       3.234254E+02       0.000000E+00       5.407499E+04       0.000000E+00        6

                                                                  COLUMN      6
      1        1.457107E+02       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00        6

                                                                  COLUMN      7
      1        0.000000E+00       0.000000E+00       1.650648E+02       0.000000E+00       1.433457E+04       0.000000E+00        6

                                                                  COLUMN      8
      1        6.657350E+01       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00        6

                                                                  COLUMN      9
      1        3.427222E+01       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00        6

                                                                  COLUMN     10
      1        1.823635E+01       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00        6

                                                                  COLUMN     11
      1        0.000000E+00       0.000000E+00       9.941160E+01       0.000000E+00       5.380285E+03       0.000000E+00        6

                                                                  COLUMN     12
      1        9.388121E+00       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00        6

                                                                  COLUMN     13
      1        4.289322E+00       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00        6

                                                                  COLUMN     14
      1        1.440945E+00       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00        6

                                                                  COLUMN     15
      1        1.548490E-01       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00       0.000000E+00        6

                                                                  COLUMN     16
      1        0.000000E+00       0.000000E+00       6.574519E+01       0.000000E+00       2.489824E+03       0.000000E+00        6
```

INTERMEDIATE MATRIX ... EFFMASSO

```
                                             COLUMN     17
1         0.000000E+00       0.000000E+00       4.543563E+01       0.000000E+00       1.312781E+03       0.000000E+00         6

                                             COLUMN     18
1         0.000000E+00       0.000000E+00       3.082773E+01       0.000000E+00       7.303151E+02       0.000000E+00         6

                                             COLUMN     19
1         0.000000E+00       0.000000E+00       1.792430E+01       0.000000E+00       3.713105E+02       0.000000E+00         6

                                             COLUMN     20
1         0.000000E+00       0.000000E+00       5.852262E+00       0.000000E+00       1.123919E+02       0.000000E+00         6

                                             COLUMN     21
1         0.000000E+00       0.000000E+00       0.000000E+00       8.479251E+00       0.000000E+00       0.000000E+00         6
```

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    DMAP FOR MODAL PART. FACTORS AND EFFEC. MASS FOR BASE MOTION.         NOVEMBER  20, 2000  MSC/NASTRAN 10/22/98   PAGE    36
     CANTILEVERED BEAM

^^^**************************************************   **************************************************
 ^^^MASSSUM IS SUM OF EFFECTIVE MASSES IN EFFMASSO.  UNITS SAME AS THOSE IN GRID POINT WEIGHT GENERATOR
 ^^^**************************************************   **************************************************
                                                  INTERMEDIATE MATRIX ... MASSSUM

                                                            COLUMN      1
    1         4.750000E+03       0.000000E+00       4.750000E+03       8.479251E+00       1.675000E+07       0.000000E+00        6



 ^^^**************************************************   **************************************************
 ^^^PMASSSUM IS SUM OF EFFECTIVE MASSES IN EFFMASSO AS A PERCENT OF THE TOTAL RB MASS (WEIGHT)
    MATRIX DIAGONAL (CALCULATED RELATIVE TO MPFPNT). THIS IS UNITLESS - NUMBERS ARE %
 ^^^**************************************************   **************************************************
    DMAP FOR MODAL PART. FACTORS AND EFFEC. MASS FOR BASE MOTION.         NOVEMBER  20, 2000  MSC/NASTRAN 10/22/98   PAGE    37
     CANTILEVERED BEAM

                                                  INTERMEDIATE MATRIX ... PMASSSUM

                                                            COLUMN      1
    1         9.500000E+01       0.000000E+00       9.500000E+01       7.708410E+01       1.000000E+02       0.000000E+00        6
```

CANTILEVERED BEAM

EIGENVALUE = 3.782232E+02
CYCLES = 3.095239E+00 R E A L E I G E N V E C T O R N O . 1

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	.0	.0	1.000000E+00	.0	-1.380306E-02	.0
2	G	.0	.0	8.620715E-01	.0	-1.377244E-02	.0
3	G	.0	.0	7.249314E-01	.0	-1.362777E-02	.0
4	G	.0	.0	5.902202E-01	.0	-1.327184E-02	.0
5	G	.0	.0	4.604671E-01	.0	-1.262412E-02	.0
6	G	.0	.0	3.389251E-01	.0	-1.162024E-02	.0
7	G	.0	.0	2.294127E-01	.0	-1.021124E-02	.0
8	G	.0	.0	1.361666E-01	.0	-8.362321E-03	.0
9	G	.0	.0	6.370739E-02	.0	-6.051082E-03	.0
10	G	.0	.0	1.672684E-02	.0	-3.265284E-03	.0
11	G	.0	.0	.0	.0	.0	.0

EIGENVALUE = 9.503404E+03
CYCLES = 1.551528E+01 R E A L E I G E N V E C T O R N O . 2

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	1.000000E+00	.0	.0	.0	.0	.0
2	G	9.876884E-01	.0	.0	.0	.0	.0
3	G	9.510565E-01	.0	.0	.0	.0	.0
4	G	8.910065E-01	.0	.0	.0	.0	.0
5	G	8.090170E-01	.0	.0	.0	.0	.0
6	G	7.071068E-01	.0	.0	.0	.0	.0
7	G	5.877852E-01	.0	.0	.0	.0	.0
8	G	4.539905E-01	.0	.0	.0	.0	.0
9	G	3.090170E-01	.0	.0	.0	.0	.0
10	G	1.564345E-01	.0	.0	.0	.0	.0
11	G	.0	.0	.0	.0	.0	.0

EIGENVALUE = 1.452556E+04
CYCLES = 1.918167E+01 R E A L E I G E N V E C T O R N O . 3

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	.0	.0	1.000000E+00	.0	-4.920211E-02	.0
2	G	.0	.0	5.118992E-01	.0	-4.802600E-02	.0
3	G	.0	.0	5.133449E-02	.0	-4.329354E-02	.0
4	G	.0	.0	-3.377010E-01	.0	-3.367990E-02	.0
5	G	.0	.0	-6.082380E-01	.0	-1.985865E-02	.0
6	G	.0	.0	-7.282655E-01	.0	-4.054879E-03	.0
7	G	.0	.0	-6.932064E-01	.0	1.058765E-02	.0
8	G	.0	.0	-5.315288E-01	.0	2.072531E-02	.0
9	G	.0	.0	-3.033195E-01	.0	2.347723E-02	.0
10	G	.0	.0	-9.314925E-02	.0	1.687965E-02	.0
11	G	.0	.0	.0	.0	.0	.0

EIGENVALUE = 8.413237E+04
CYCLES = 4.616381E+01 R E A L E I G E N V E C T O R N O . 4

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	1.000000E+00	.0	.0	.0	.0	.0
2	G	8.910065E-01	.0	.0	.0	.0	.0
3	G	5.877852E-01	.0	.0	.0	.0	.0
4	G	1.564345E-01	.0	.0	.0	.0	.0
5	G	-3.090170E-01	.0	.0	.0	.0	.0
6	G	-7.071068E-01	.0	.0	.0	.0	.0
7	G	-9.510565E-01	.0	.0	.0	.0	.0
8	G	-9.876884E-01	.0	.0	.0	.0	.0
9	G	-8.090170E-01	.0	.0	.0	.0	.0
10	G	-4.539905E-01	.0	.0	.0	.0	.0
11	G	.0	.0	.0	.0	.0	.0

CANTILEVERED BEAM

EIGENVALUE = 1.116134E+05
CYCLES = 5.317143E+01 R E A L E I G E N V E C T O R N O . 5

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	.0	.0	1.000000E+00	.0	-8.548352E-02	.0
2	G	.0	.0	1.752888E-01	.0	-7.644632E-02	.0
3	G	.0	.0	-4.581177E-01	.0	-4.616648E-02	.0
4	G	.0	.0	-6.942720E-01	.0	2.440367E-04	.0
5	G	.0	.0	-4.688972E-01	.0	4.195648E-02	.0
6	G	.0	.0	5.911879E-02	.0	5.794726E-02	.0
7	G	.0	.0	5.796209E-01	.0	4.080989E-02	.0
8	G	.0	.0	8.033721E-01	.0	2.089192E-03	.0
9	G	.0	.0	6.327829E-01	.0	-3.321805E-02	.0
10	G	.0	.0	2.369147E-01	.0	-3.915421E-02	.0
11	G	.0	.0	.0	.0	.0	.0

EIGENVALUE = 2.260851E+05
CYCLES = 7.567564E+01 R E A L E I G E N V E C T O R N O . 6

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	-1.000000E+00	.0	.0	.0	.0	.0
2	G	-7.071068E-01	.0	.0	.0	.0	.0
3	G	2.484774E-16	.0	.0	.0	.0	.0
4	G	7.071068E-01	.0	.0	.0	.0	.0
5	G	1.000000E+00	.0	.0	.0	.0	.0
6	G	7.071068E-01	.0	.0	.0	.0	.0
7	G	-7.753027E-18	.0	.0	.0	.0	.0
8	G	-7.071068E-01	.0	.0	.0	.0	.0
9	G	-1.000000E+00	.0	.0	.0	.0	.0
10	G	-7.071068E-01	.0	.0	.0	.0	.0
11	G	.0	.0	.0	.0	.0	.0

CANTILEVERED BEAM

EIGENVALUE = 4.197146E+05
CYCLES = 1.031091E+02 R E A L E I G E N V E C T O R N O . 7

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	.0	.0	1.000000E+00	.0	-1.302853E-01	.0
2	G	.0	.0	-1.895739E-01	.0	-9.630155E-02	.0
3	G	.0	.0	-7.424221E-01	.0	-7.235121E-03	.0
4	G	.0	.0	-3.618190E-01	.0	7.356850E-02	.0
5	G	.0	.0	4.512300E-01	.0	7.105675E-02	.0
6	G	.0	.0	8.018550E-01	.0	-8.693301E-03	.0
7	G	.0	.0	3.037999E-01	.0	-8.051258E-02	.0
8	G	.0	.0	-5.314652E-01	.0	-6.925244E-02	.0
9	G	.0	.0	-8.558970E-01	.0	9.613269E-03	.0
10	G	.0	.0	-4.281665E-01	.0	6.178896E-02	.0
11	G	.0	.0	.0	.0	.0	.0

EIGENVALUE = 4.214662E+05
CYCLES = 1.033241E+02 R E A L E I G E N V E C T O R N O . 8

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	1.000000E+00	.0	.0	.0	.0	.0
2	G	4.539905E-01	.0	.0	.0	.0	.0
3	G	-5.877852E-01	.0	.0	.0	.0	.0
4	G	-9.876884E-01	.0	.0	.0	.0	.0
5	G	-3.090170E-01	.0	.0	.0	.0	.0
6	G	7.071068E-01	.0	.0	.0	.0	.0
7	G	9.510565E-01	.0	.0	.0	.0	.0
8	G	1.564345E-01	.0	.0	.0	.0	.0
9	G	-8.090170E-01	.0	.0	.0	.0	.0
10	G	-8.910065E-01	.0	.0	.0	.0	.0
11	G	.0	.0	.0	.0	.0	.0

EIGENVALUE = 6.511506E+05
CYCLES = 1.284283E+02 R E A L E I G E N V E C T O R N O . 9

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	1.000000E+00	.0	.0	.0	.0	.0
2	G	1.564345E-01	.0	.0	.0	.0	.0
3	G	-9.510565E-01	.0	.0	.0	.0	.0
4	G	-4.539905E-01	.0	.0	.0	.0	.0
5	G	8.090170E-01	.0	.0	.0	.0	.0
6	G	7.071068E-01	.0	.0	.0	.0	.0
7	G	-5.877852E-01	.0	.0	.0	.0	.0
8	G	-8.910065E-01	.0	.0	.0	.0	.0
9	G	3.090170E-01	.0	.0	.0	.0	.0
10	G	9.876884E-01	.0	.0	.0	.0	.0
11	G	.0	.0	.0	.0	.0	.0

EIGENVALUE = 8.926549E+05
CYCLES = 1.503703E+02 R E A L E I G E N V E C T O R N O . 10

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	1.000000E+00	.0	.0	.0	.0	.0
2	G	-1.564345E-01	.0	.0	.0	.0	.0
3	G	-9.510565E-01	.0	.0	.0	.0	.0
4	G	4.539905E-01	.0	.0	.0	.0	.0
5	G	8.090170E-01	.0	.0	.0	.0	.0
6	G	-7.071068E-01	.0	.0	.0	.0	.0
7	G	-5.877852E-01	.0	.0	.0	.0	.0
8	G	8.910065E-01	.0	.0	.0	.0	.0
9	G	3.090170E-01	.0	.0	.0	.0	.0
10	G	-9.876884E-01	.0	.0	.0	.0	.0
11	G	.0	.0	.0	.0	.0	.0

CANTILEVERED BEAM

EIGENVALUE = 1.121570E+06
CYCLES = 1.685517E+02   R E A L   E I G E N V E C T O R   N O .   11

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	.0	.0	1.000000E+00	.0	-1.865075E-01	.0
2	G	.0	.0	-5.623685E-01	.0	-9.569546E-02	.0
3	G	.0	.0	-6.489609E-01	.0	7.460109E-02	.0
4	G	.0	.0	4.612461E-01	.0	1.043754E-01	.0
5	G	.0	.0	8.564952E-01	.0	-4.046599E-02	.0
6	G	.0	.0	-1.323474E-01	.0	-1.205895E-01	.0
7	G	.0	.0	-9.011596E-01	.0	-4.472405E-03	.0
8	G	.0	.0	-1.933596E-01	.0	1.201755E-01	.0
9	G	.0	.0	8.681486E-01	.0	5.456298E-02	.0
10	G	.0	.0	6.722254E-01	.0	-7.875189E-02	.0
11	G	.0	.0	.0	.0	.0	.0

EIGENVALUE = 1.122339E+06
CYCLES = 1.686096E+02   R E A L   E I G E N V E C T O R   N O .   12

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	1.000000E+00	.0	.0	.0	.0	.0
2	G	-4.539905E-01	.0	.0	.0	.0	.0
3	G	-5.877852E-01	.0	.0	.0	.0	.0
4	G	9.876884E-01	.0	.0	.0	.0	.0
5	G	-3.090170E-01	.0	.0	.0	.0	.0
6	G	-7.071068E-01	.0	.0	.0	.0	.0
7	G	9.510565E-01	.0	.0	.0	.0	.0
8	G	-1.564345E-01	.0	.0	.0	.0	.0
9	G	-8.090170E-01	.0	.0	.0	.0	.0
10	G	8.910065E-01	.0	.0	.0	.0	.0
11	G	.0	.0	.0	.0	.0	.0

EIGENVALUE = 1.317720E+06
CYCLES = 1.826971E+02   R E A L   E I G E N V E C T O R   N O .   13

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	1.000000E+00	.0	.0	.0	.0	.0
2	G	-7.071068E-01	.0	.0	.0	.0	.0
3	G	2.775024E-17	.0	.0	.0	.0	.0
4	G	7.071068E-01	.0	.0	.0	.0	.0
5	G	-1.000000E+00	.0	.0	.0	.0	.0
6	G	7.071068E-01	.0	.0	.0	.0	.0
7	G	-1.678946E-16	.0	.0	.0	.0	.0
8	G	-7.071068E-01	.0	.0	.0	.0	.0
9	G	1.000000E+00	.0	.0	.0	.0	.0
10	G	-7.071068E-01	.0	.0	.0	.0	.0
11	G	.0	.0	.0	.0	.0	.0

EIGENVALUE = 1.459673E+06
CYCLES = 1.922861E+02   R E A L   E I G E N V E C T O R   N O .   14

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	1.000000E+00	.0	.0	.0	.0	.0
2	G	-8.910065E-01	.0	.0	.0	.0	.0
3	G	5.877852E-01	.0	.0	.0	.0	.0
4	G	-1.564345E-01	.0	.0	.0	.0	.0
5	G	-3.090170E-01	.0	.0	.0	.0	.0
6	G	7.071068E-01	.0	.0	.0	.0	.0
7	G	-9.510565E-01	.0	.0	.0	.0	.0
8	G	9.876884E-01	.0	.0	.0	.0	.0
9	G	-8.090170E-01	.0	.0	.0	.0	.0
10	G	4.539905E-01	.0	.0	.0	.0	.0
11	G	.0	.0	.0	.0	.0	.0

EIGENVALUE = 1.534302E+06
CYCLES = 1.971404E+02 R E A L E I G E N V E C T O R N O . 15

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	1.000000E+00	.0	.0	.0	.0	.0
2	G	-9.876884E-01	.0	.0	.0	.0	.0
3	G	9.510565E-01	.0	.0	.0	.0	.0
4	G	-8.910065E-01	.0	.0	.0	.0	.0
5	G	8.090170E-01	.0	.0	.0	.0	.0
6	G	-7.071068E-01	.0	.0	.0	.0	.0
7	G	5.877852E-01	.0	.0	.0	.0	.0
8	G	-4.539905E-01	.0	.0	.0	.0	.0
9	G	3.090170E-01	.0	.0	.0	.0	.0
10	G	-1.564345E-01	.0	.0	.0	.0	.0
11	G	.0	.0	.0	.0	.0	.0

EIGENVALUE = 2.435699E+06
CYCLES = 2.483888E+02 R E A L E I G E N V E C T O R N O . 16

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	.0	.0	9.263998E-01	.0	-2.394046E-01	.0
2	G	.0	.0	-8.586448E-01	.0	-5.670418E-02	.0
3	G	.0	.0	-1.186018E-01	.0	1.527209E-01	.0
4	G	.0	.0	1.000000E+00	.0	3.414156E-03	.0
5	G	.0	.0	-8.725514E-02	.0	-1.569735E-01	.0
6	G	.0	.0	-9.763568E-01	.0	3.157281E-02	.0
7	G	.0	.0	3.089091E-01	.0	1.495315E-01	.0
8	G	.0	.0	9.014412E-01	.0	-6.635948E-02	.0
9	G	.0	.0	-5.397807E-01	.0	-1.387005E-01	.0
10	G	.0	.0	-9.185680E-01	.0	7.515867E-02	.0
11	G	.0	.0	.0	.0	.0	.0

EIGENVALUE = 4.563269E+06
CYCLES = 3.399837E+02 R E A L E I G E N V E C T O R N O . 17

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	.0	.0	7.008497E-01	.0	-2.448726E-01	.0
2	G	.0	.0	-8.847049E-01	.0	1.407885E-02	.0
3	G	.0	.0	5.295517E-01	.0	1.371678E-01	.0
4	G	.0	.0	5.309585E-01	.0	-1.380512E-01	.0
5	G	.0	.0	-9.469009E-01	.0	-2.789894E-02	.0
6	G	.0	.0	2.329952E-01	.0	1.602579E-01	.0
7	G	.0	.0	7.597703E-01	.0	-1.011314E-01	.0
8	G	.0	.0	-8.427259E-01	.0	-7.844863E-02	.0
9	G	.0	.0	-6.663252E-02	.0	1.670051E-01	.0
10	G	.0	.0	1.000000E+00	.0	-3.675383E-02	.0
11	G	.0	.0	.0	.0	.0	.0

EIGENVALUE = 7.536524E+06
CYCLES = 4.369238E+02 R E A L E I G E N V E C T O R N O . 18

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	.0	.0	-4.924617E-01	.0	2.255767E-01	.0
2	G	.0	.0	7.616004E-01	.0	-7.493474E-02	.0
3	G	.0	.0	-8.962593E-01	.0	-4.697701E-02	.0
4	G	.0	.0	3.691444E-01	.0	1.451060E-01	.0
5	G	.0	.0	4.085605E-01	.0	-1.420010E-01	.0
6	G	.0	.0	-9.043526E-01	.0	4.084879E-02	.0
7	G	.0	.0	7.751005E-01	.0	8.856782E-02	.0
8	G	.0	.0	-1.092490E-01	.0	-1.565890E-01	.0
9	G	.0	.0	-6.241466E-01	.0	1.180141E-01	.0
10	G	.0	.0	1.000000E+00	.0	1.730748E-02	.0
11	G	.0	.0	.0	.0	.0	.0

EIGENVALUE = 1.093716E+07
CYCLES = 5.263474E+02 R E A L E I G E N V E C T O R N O . 19

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	.0	.0	-3.384672E-01	.0	1.932924E-01	.0
2	G	.0	.0	5.953377E-01	.0	-1.064434E-01	.0
3	G	.0	.0	-9.508262E-01	.0	4.877345E-02	.0
4	G	.0	.0	9.885869E-01	.0	2.932435E-02	.0
5	G	.0	.0	-7.236277E-01	.0	-9.791131E-02	.0
6	G	.0	.0	2.353350E-01	.0	1.363453E-01	.0
7	G	.0	.0	3.254764E-01	.0	-1.327388E-01	.0
8	G	.0	.0	-7.863307E-01	.0	8.811001E-02	.0
9	G	.0	.0	1.000000E+00	.0	-1.734421E-02	.0
10	G	.0	.0	-9.594333E-01	.0	-7.066394E-02	.0
11	G	.0	.0	.0	.0	.0	.0

EIGENVALUE = 1.373947E+07
CYCLES = 5.899363E+02 R E A L E I G E N V E C T O R N O . 20

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	.0	.0	1.623984E-01	.0	-1.070985E-01	.0
2	G	.0	.0	-3.063766E-01	.0	7.356451E-02	.0
3	G	.0	.0	5.658817E-01	.0	-6.611455E-02	.0
4	G	.0	.0	-7.718680E-01	.0	5.124629E-02	.0
5	G	.0	.0	9.199195E-01	.0	-3.265926E-02	.0
6	G	.0	.0	-9.979247E-01	.0	1.157372E-02	.0
7	G	.0	.0	1.000000E+00	.0	1.038852E-02	.0
8	G	.0	.0	-9.261574E-01	.0	-3.159759E-02	.0
9	G	.0	.0	7.796957E-01	.0	4.991053E-02	.0
10	G	.0	.0	-6.025249E-01	.0	-7.095481E-02	.0
11	G	.0	.0	.0	.0	.0	.0

CANTILEVERED BEAM

EIGENVALUE = 2.652774E+08
CYCLES = 2.592210E+03   R E A L   E I G E N V E C T O R   N O .   21

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	.0	.0	.0	1.000000E+00	.0	.0
2	G	.0	.0	.0	9.776617E-01	.0	.0
3	G	.0	.0	.0	9.334840E-01	.0	.0
4	G	.0	.0	.0	8.684538E-01	.0	.0
5	G	.0	.0	.0	7.840238E-01	.0	.0
6	G	.0	.0	.0	6.820800E-01	.0	.0
7	G	.0	.0	.0	5.648996E-01	.0	.0
8	G	.0	.0	.0	4.351004E-01	.0	.0
9	G	.0	.0	.0	2.955817E-01	.0	.0
10	G	.0	.0	.0	1.494602E-01	.0	.0
11	G	.0	.0	.0	.0	.0	.0