DCATT Jitter Analysis using an Integrated Modeling Approach

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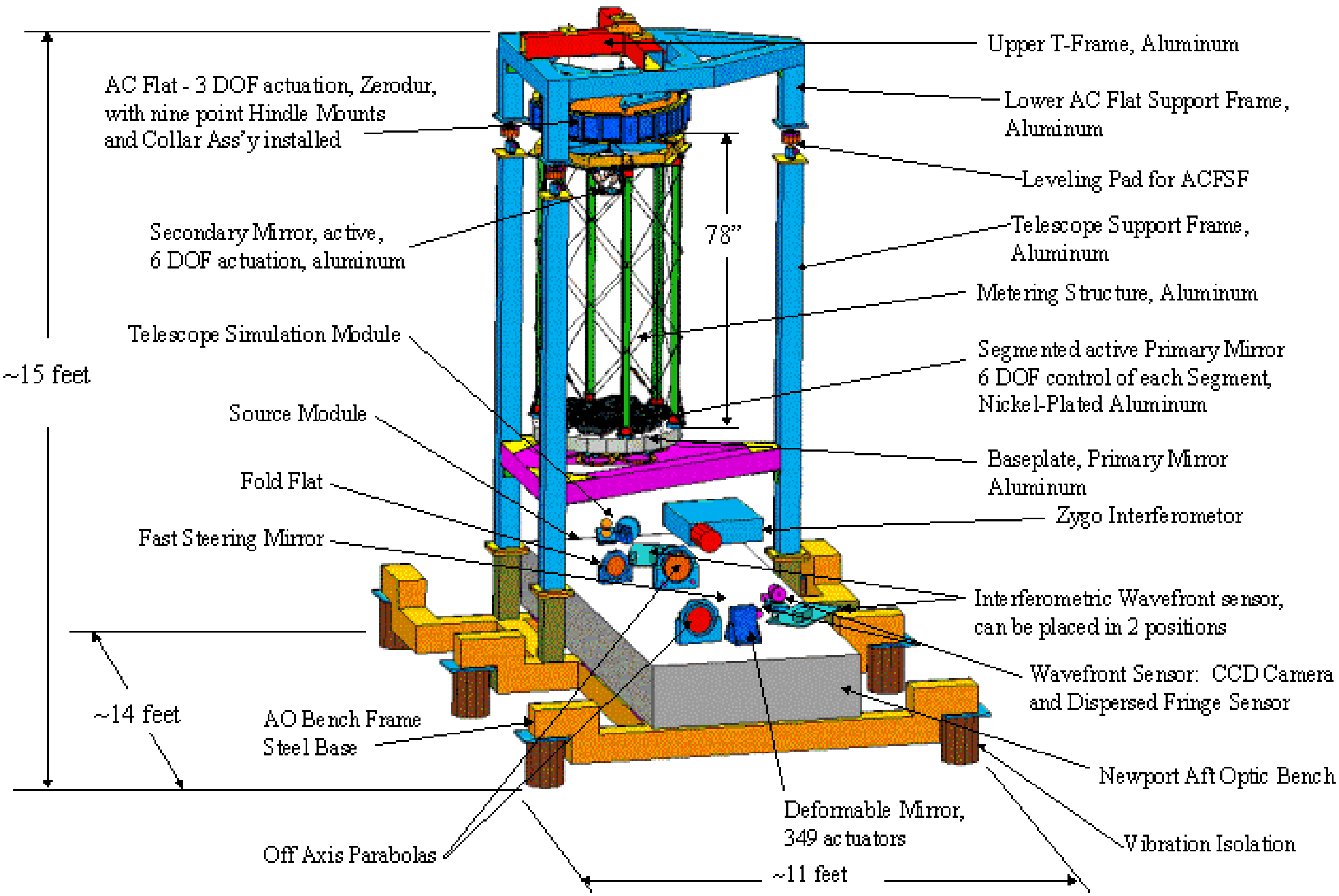
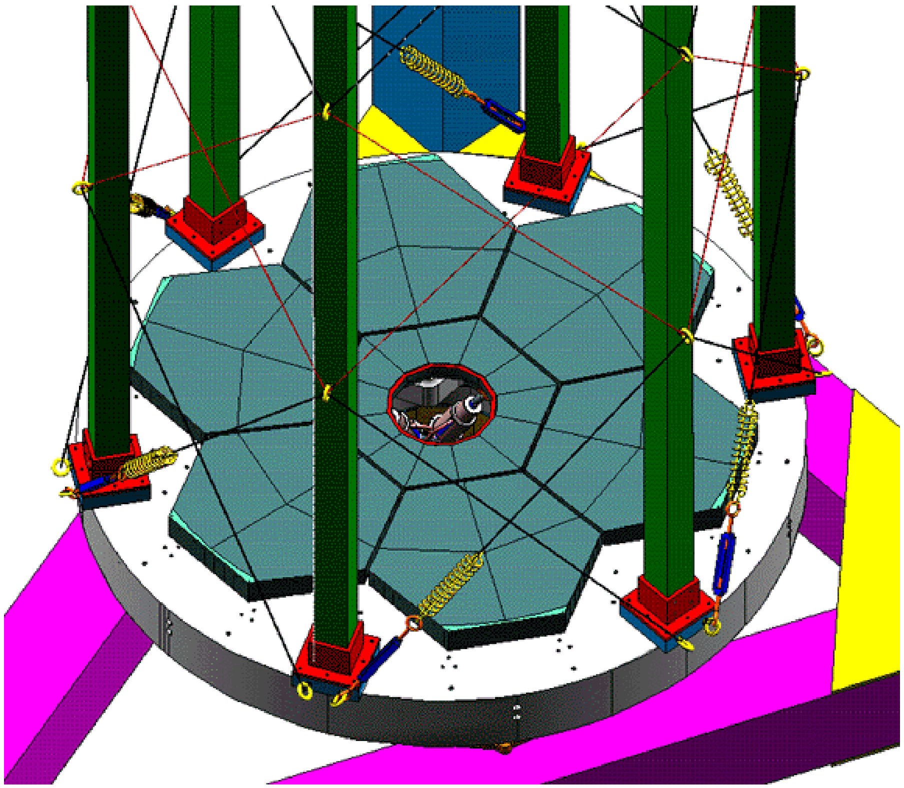
Code 731

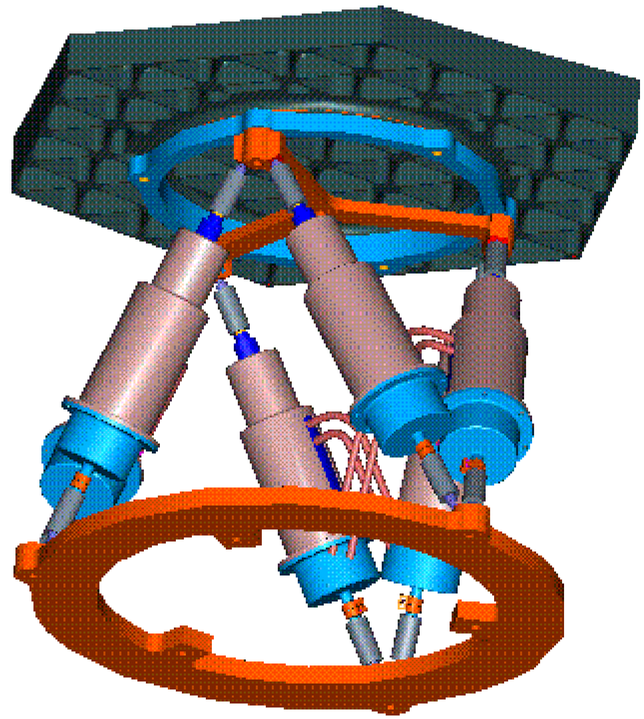
What DCATT Was Supposed To Be



Additional Views



Primary Mirror Closeup



Hexapod Details

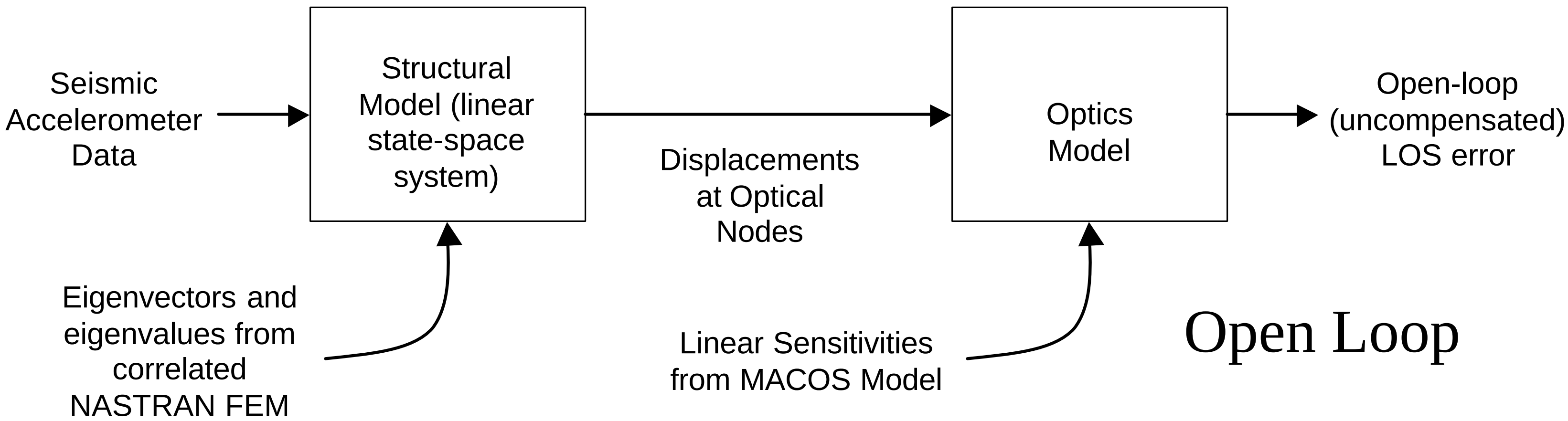
Analysis Objectives

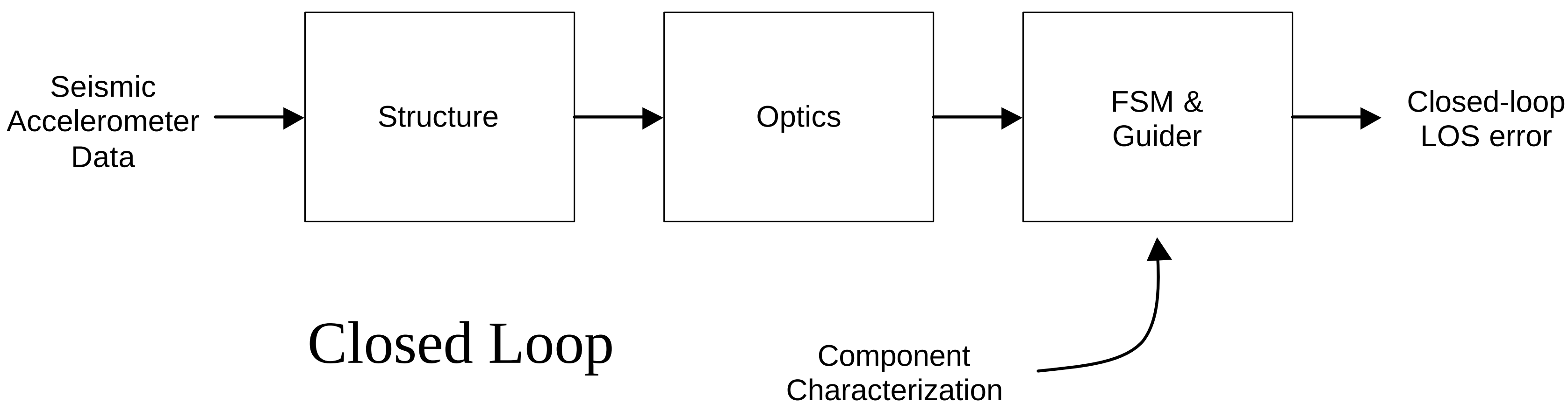
- Combine models of the testbed structure and optics, driven by external loads measured in the lab environment, to predict the uncompensated image motion on the focal plane
- Verify the models against measured jitter and update as required
- Derive design requirements for an image stabilization subsystem (FSM loop)
- Extend the model to include the FSM loop
- Verify the models against the measurements on final testbed configuration and update as required

Modeling Approach

- Employ linear systems theory, enabling analysis to proceed in both time and frequency domains
- Matlab used to construct systems-level model
- Optics model parameters from MACOS sensitivity analysis
- Dynamics state-space model parameters from NASTRAN normal modes analysis

Integrated Modeling Framework





Open-Loop Linear Systems Model

$$\dot{\mathbf{X}} = \mathbf{A}_1\mathbf{X} + \mathbf{B}_1\mathbf{U}$$

$$\mathbf{Y} = \mathbf{K}\mathbf{C}_1\mathbf{X}$$

- **X** is the vector of generalized coordinates and their first-deriviatives (velocities)
- **U** is the vector of input (base) forces **(note: requires seismic mass in model)**
- **Y** is the vector of LOS errors
- $\mathbf{A_1}$ contains the natural frequencies and modal damping for the structure
- $\mathbf{B_1}$ contains the mode shapes for the input node
- $\mathbf{C_1}$ contains the mode shapes for the output nodes (note that for this system there is no $\mathbf{D_1}$, or feed-through, term)
- **K** contains the optical sensitivites

Closed-Loop Linear Systems Model

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{BU}$$

$$\mathbf{Y} = \mathbf{CX}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{B}_1\mathbf{C}_2 \\ \mathbf{0} & \mathbf{A}_2 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{B}_2 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} \mathbf{KC}_1 & \mathbf{0} \end{bmatrix}$$

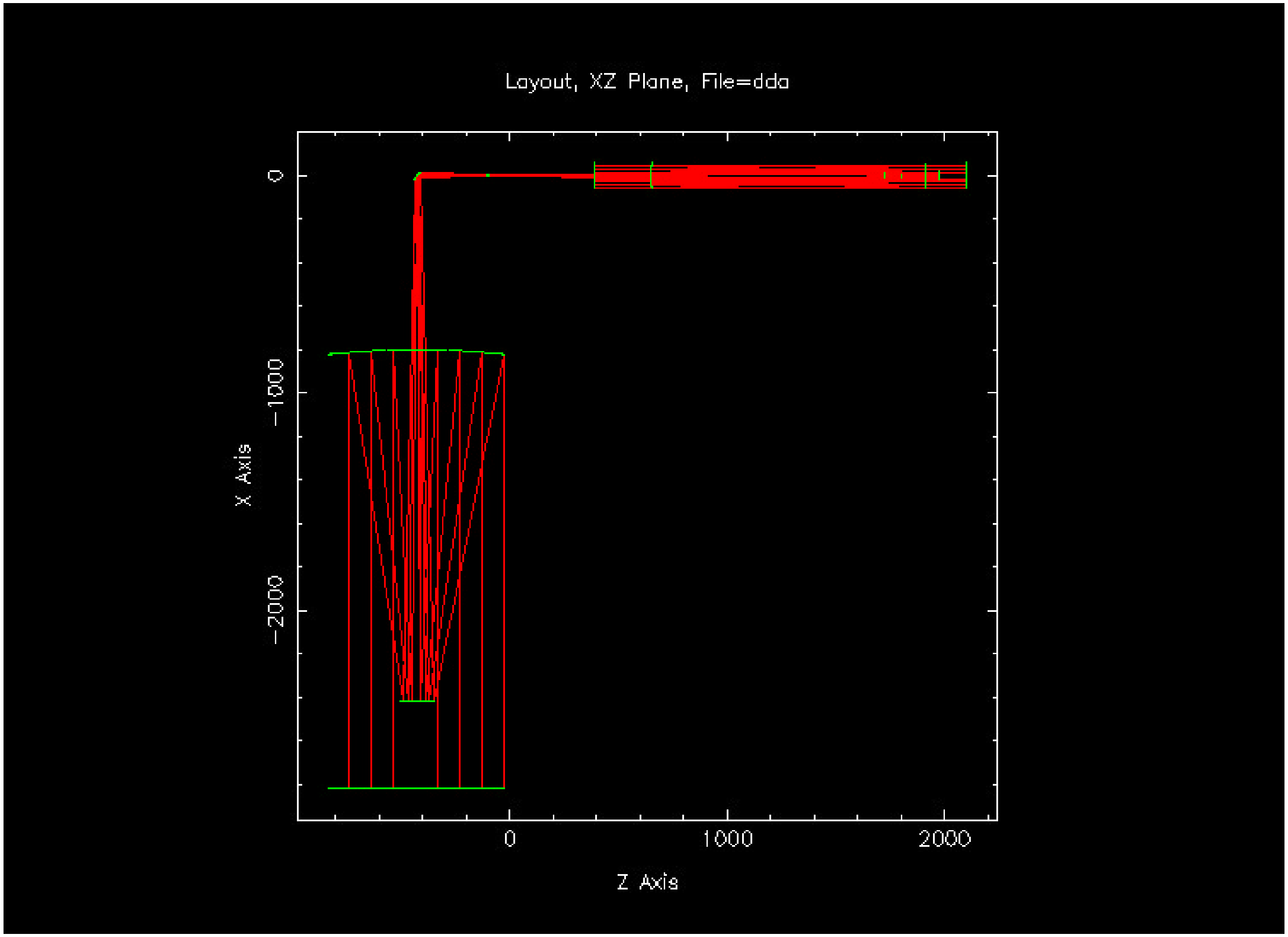
- **X** is the vector of generalized coordinates and their first-deriviatives (velocities), augmented by additional states for the FSM plant and controller
- $\mathbf{A_2}$, $\mathbf{B_2}$, $\mathbf{C_2}$ comprise the state-space model for the FSM plant and controller

Optics Modeling

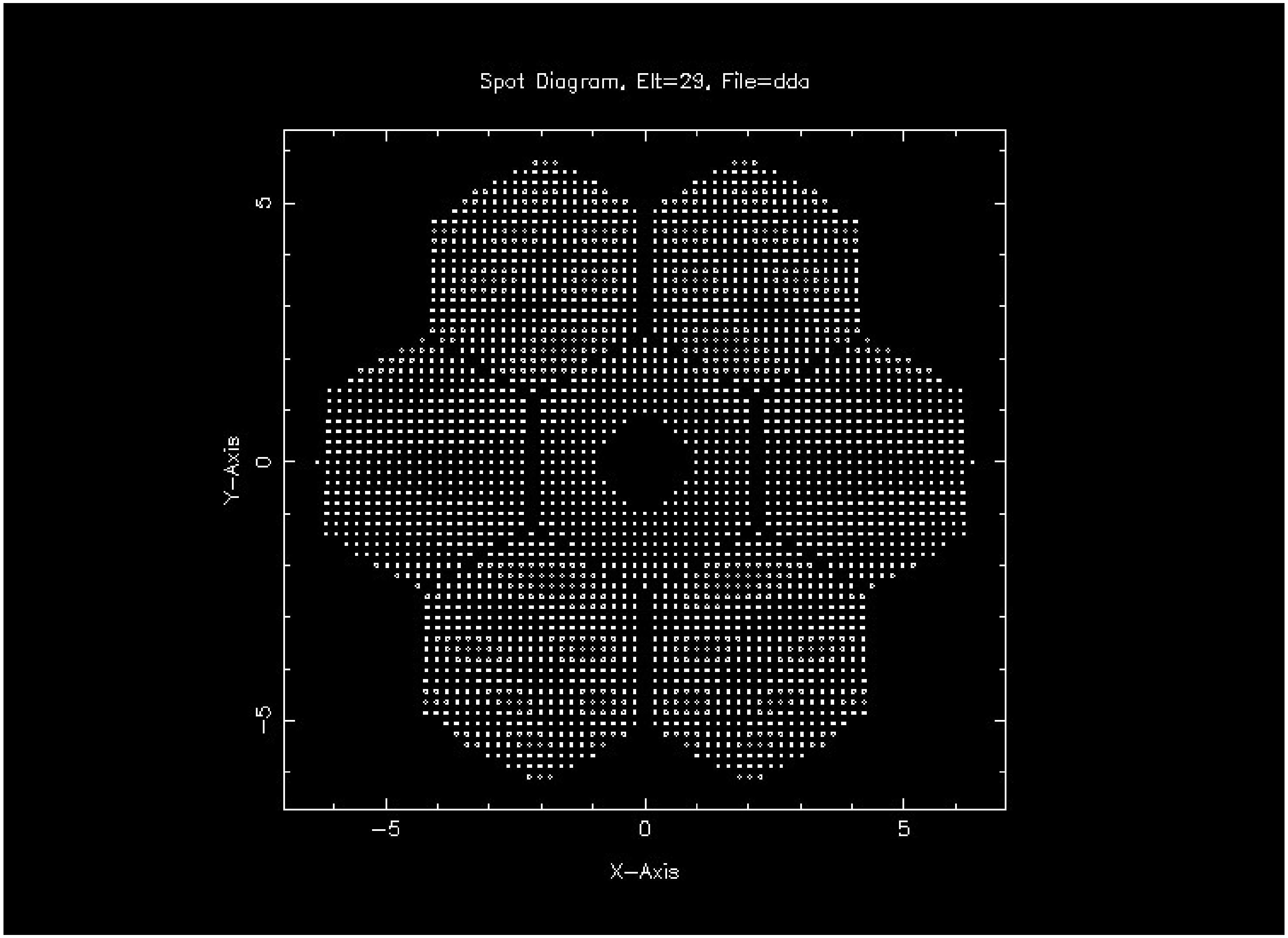
- Time domain analysis could utilize a full non-linear ray trace capability of MACOS
- Frequency domain analysis requires a linear model (sensitivity matrix)
- Sensitivity matrix obtained by introducing small kinematic perturbations to each DOF of each optic, one DOF at a time (20 optical elements, 6 kinematic DOF each, total of 120 DOF), running full ray trace, observing centroid shift on focal plane in each of 2 coordinates, and dividing delta-centroid by delta-translation/rotation
- K matrix for this model is 2x120

$$\Theta(t) = \mathbf{Ku}(t)$$

MACOS Ray Trace Model

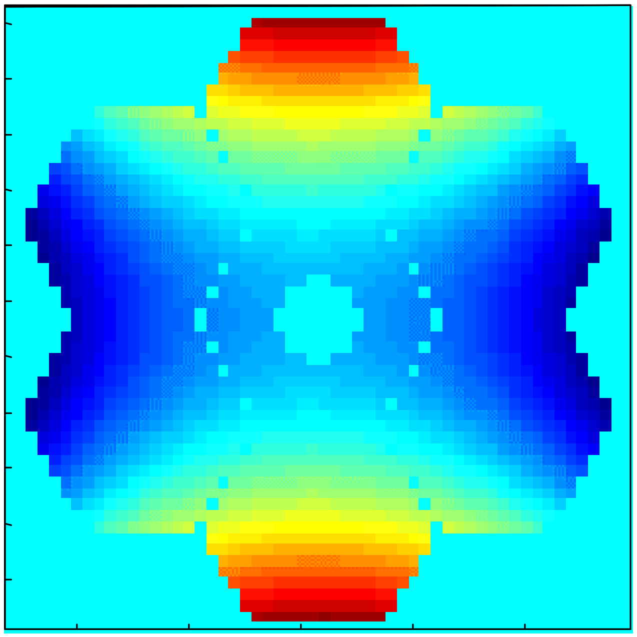


Spot Diagram at Entrance Pupil



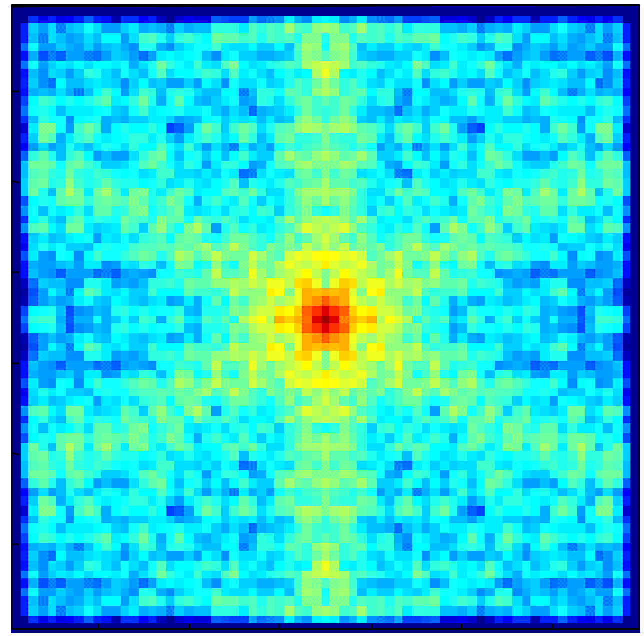
Nominal OPD and PSF

Nominal OPD



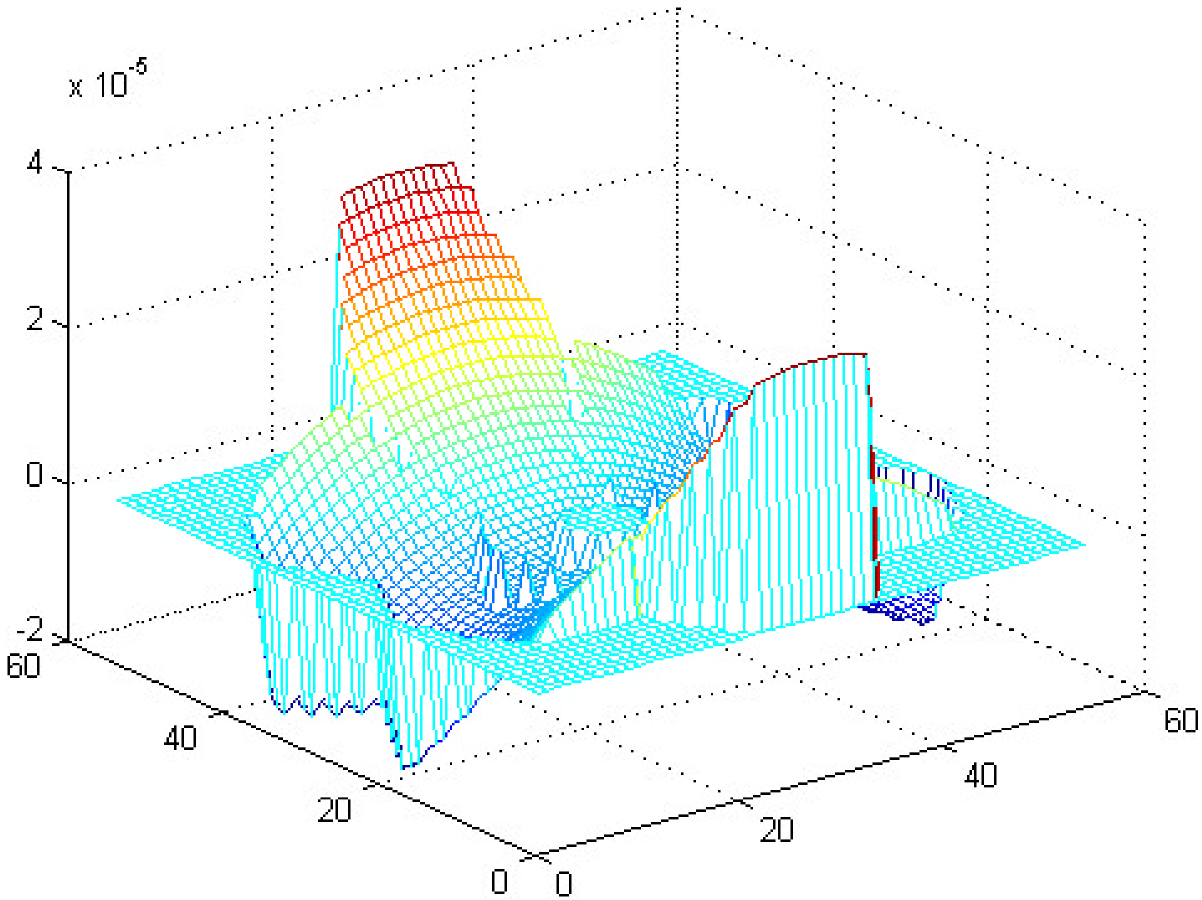
WFE = 0.00050655 microns

Nominal PSF (log stretch)

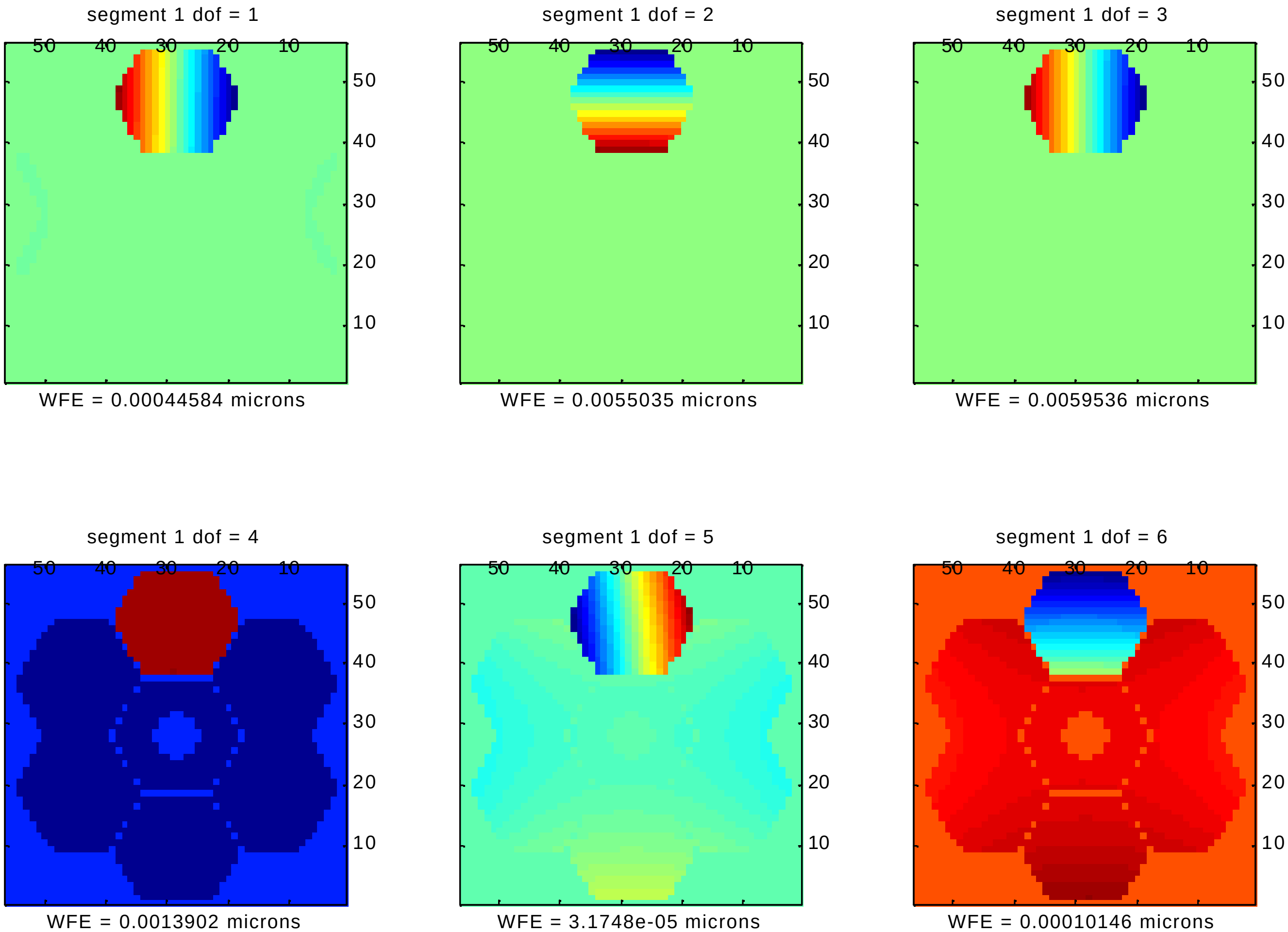


LOS = 0 microns

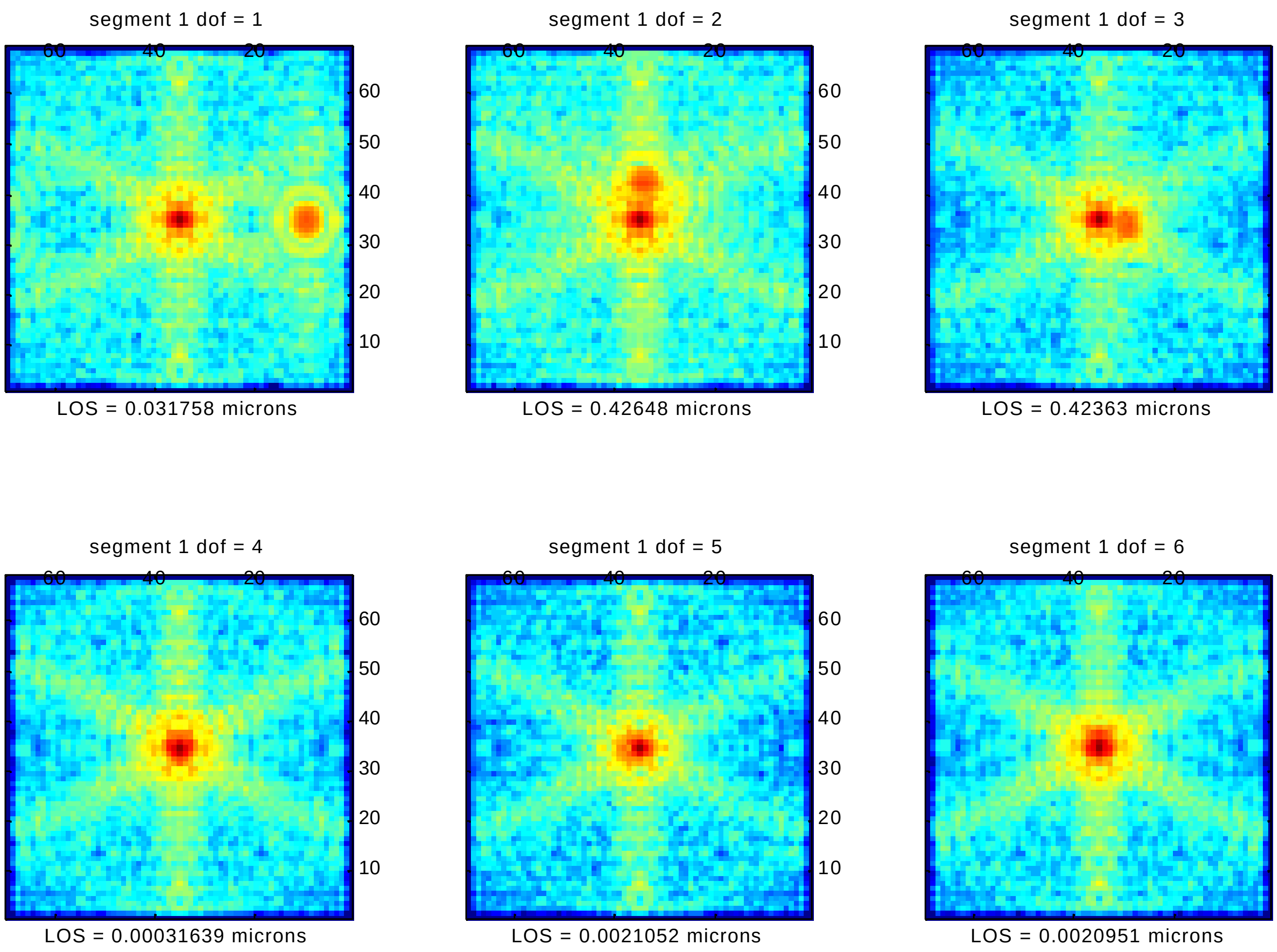
Nominal OPD



Perturbed OPD



Perturbed PSF



Optical Sensitivities, 1 of 4

Element	MACOS (mm/mm or mm/rad)		Wilson (converted)	Wilson
	dx	dy	mm/mm or mm/rad	9
PM1	1.5042E+00	6.3515E+02		
	-8.5296E+03	1.9124E+01	-8.5585E+03	-32.272
	1.9736E+01	8.4727E+03		
	-3.1639E-01	7.4433E-04		
	-4.9676E-03	-2.1052E+00		
	-2.0951E+00	5.1173E-03	-2.1396E+00	-1.6641
PM2	5.7689E+02	3.3124E+02		
	-8.8900E+03	4.2094E+01		
	-1.8838E+00	8.9096E+03		
	-1.6540E-01	2.8954E-01		
	-1.5647E-02	-2.1885E+00		
	-2.2114E+00	-5.3964E-03		
PM3	5.7536E+02	-3.3365E+02		
	-8.8897E+03	-4.0163E+00		
	4.2797E+01	8.9095E+03		
	1.6540E-01	2.8954E-01		
	5.4408E-03	-2.1885E+00		
	-2.2114E+00	1.5631E-02		
PM4	-1.4246E+00	-6.3495E+02		
	-8.5289E+03	1.6939E+01		
	1.9189E+01	8.4727E+03		
	3.1639E-01	7.4433E-04		
	-4.7015E-03	-2.1051E+00		
	-2.0951E+00	4.9312E-03		
PM5	-5.7672E+02	-3.3090E+02		
	-8.8875E+03	4.0405E+01		
	-2.1690E+00	8.9059E+03		
	1.6538E-01	-2.8672E-01		
	-1.5504E-02	-2.1848E+00		
	-2.2108E+00	-5.4584E-03		

Comparason table created on 9-22-99

Shaded cells are the translation sensitivity, and the unshaded the rotation sensitivity.

Cell E2 contains the size of the CCD pixel size in microns.

The data give by Mark Wilson is in pixels/micron or pixels/arcsecond. The primary mirror is treated as a single piece in Mark's data. The data is divided by 7 in the conversion for the PM sensitivity, since there are 7 segments in the PM.

The axis of rotation in Mark's data is generally the z-axis in the MACOS model.

Optical Sensitivities, 2 of 4

PM6	-5.7519E+02	3.3390E+02		
	-8.8879E+03	-3.4171E+00		
	4.3072E+01	8.9058E+03		
	-1.6538E-01	-2.8672E-01		
	5.3013E-03	-2.1846E+00		
	-2.2109E+00	1.6096E-02		
PM7	0.0000E+00	0.0000E+00		
	-7.6056E+03	1.6212E+01		
	1.7550E+01	7.5993E+03		
	1.0629E-11	1.2406E-03		
	-4.3837E-03	-1.8879E+00		
	-1.8997E+00	4.7451E-03		
SM	0.0000E+00	0.0000E+00		
	1.1484E+04	-2.6893E+01	1.1454E+04	6.1698
	-2.6501E+01	-1.1477E+04		
	1.4789E-09	5.7686E-03		
	2.9816E-02	1.2949E+01		
	1.2921E+01	-2.9401E-02	1.2981E+01	1.4423
AC flat	0.0000E+00	0.0000E+00		
	3.0208E+04	-7.2531E+01	2.9955E+04	16.136
	-6.9705E+01	-3.0191E+04		
	2.5104E-09	0.0000E+00		
	0.0000E+00	0.0000E+00		
	0.0000E+00	0.0000E+00		
Fold flat	4.6362E-08	9.0655E+02		
	-1.8122E+03	2.1127E+00		
	4.1365E+00	9.0655E+02		
	-2.0135E+00	-1.8608E-04		
	4.6462E-03	0.0000E+00		
	2.0135E+00	0.0000E+00		

Optical Sensitivities, 3 of 4

BS1	5.2980E-08	-1.1207E+02		
	1.0892E+02	3.7217E-03	2.4077E+02	0.1297
	-1.9823E+01	2.4811E-03		
	0.0000E+00	0.0000E+00		
	3.4938E-09	6.3206E-02		
	1.5399E-09	3.5021E-01		
OAP1	-1.9040E-08	3.9161E+03		
	-3.9035E+03	-5.2723E-02		
	4.3899E+02	2.3570E-02		
	9.6497E-01	0.0000E+00		
	3.1795E-09	9.7842E-01	9.9360E-01	0.1104
	2.3200E-09	3.9077E-03		
Flat1	7.5385E-08	-4.0155E+03		
	3.9215E+03	1.4266E-02		
	-7.9188E+02	-1.7368E-02		
	0.0000E+00	0.0000E+00		
	2.6675E-09	9.9244E-04		
	2.6897E-09	4.9002E-03		
DM	2.0556E-08	4.1050E+03		
	-4.0190E+03	0.0000E+00	-4.0573E+03	-2.1856
	6.6654E+02	1.9229E-02		
	0.0000E+00	0.0000E+00		
	1.4363E-09	-6.2028E-04		
	2.0501E-09	-5.3964E-03		
OAP2	7.8601E-08	-4.1638E+03		
	4.1725E+03	3.5356E-02		
	-1.7608E+02	-2.4811E-03		
	1.0259E+00	0.0000E+00		
	1.2607E-09	1.0285E+00	9.9360E-01	0.1104
	2.5880E-11	-1.5457E-01		

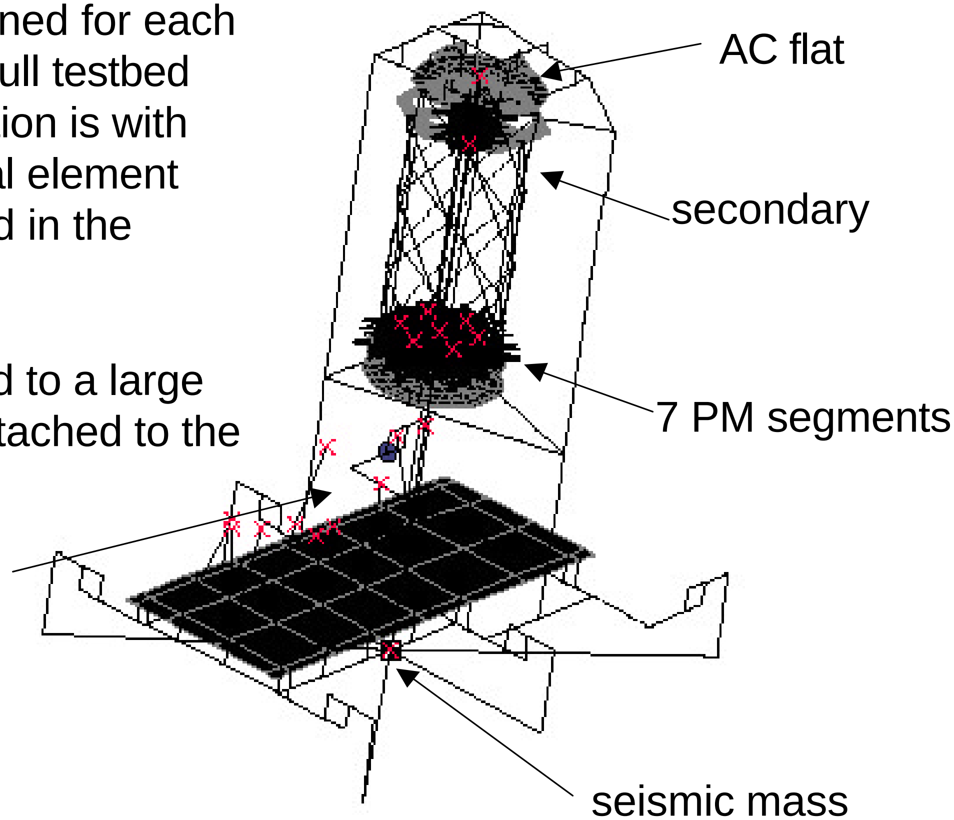
Optical Sensitivities, 4 of 4

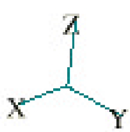
FSM	4.6806E-08	1.4785E+03		
	-1.4408E+03	1.9849E-02		
	-9.0229E+01	-1.2406E-03		
	0.0000E+00	0.0000E+00		
	2.4272E-09	-2.5803E-02		
	1.7931E-09	4.3301E-01		
BS2	4.4144E-08	-8.5815E+02		
	4.1246E+02	2.2950E-02		
	-3.1088E+02	-1.8608E-03		
	0.0000E+00	0.0000E+00		
	8.3740E-10	-9.6441E-01		
	2.4124E-09	-1.2789E+00		
Flat2	4.1001E-08	4.2972E+02		
	-2.1410E+02	6.2028E-04		
	2.2619E+02	1.3026E-02		
	0.0000E+00	0.0000E+00		
	1.8412E-09	1.0105E+00		
	1.6970E-09	9.5597E-01		
FP assy	2.3403E-08	0.0000E+00		
	-1.7743E-05	0.0000E+00		
	1.0491E-06	0.0000E+00		
	-1.0000E+00	0.0000E+00		
	0.0000E+00	-1.0000E+00		
	0.0000E+00	0.0000E+00		
Source	0.0000E+00	0.0000E+00		
	0.0000E+00	0.0000E+00		
	0.0000E+00	0.0000E+00		
	1.0226E+00	0.0000E+00		
	1.3500E-09	1.0000E+00		
	-5.0000E-10	0.0000E+00		

NASTRAN Model - Optics Nodes

For the integrated model, rigid body motion in 6-dof is obtained for each of the 20 optics in the full testbed configuration. This motion is with respect to the individual element “rotation points” defined in the MACOS model.

Input forces are applied to a large seismic mass rigidly attached to the base of the structure.





FEM coordinates

Output Nodes of FEM

iElt	EltName	Rpt (element location in MACOS coordintes)			NASTRAN Grid ID		
		X	Y	Z			
	Source				10020		
1	Beamsplitt1FR	0.0000000000E+00	-3.9610090070E-01	-1.0975096820E+02	10010		
2	TelFoldFlatR1	0.0000000000E+00	-1.1863291420E+00	-4.5220805640E+02	10011		
3	SecondaryR1	-2.4176370000E+03	-1.1863291420E+00	-4.5220805640E+02	10008		
4	PrimS1R1	-8.1125000000E+02	-1.1863291400E+00	-7.5220805640E+02	10001		
5	PrimS2R1	-8.1125000000E+02	2.5862129199E+02	-6.0220805640E+02	10002		
6	PrimS3R1	-8.1125000000E+02	2.5862129199E+02	-3.0220805640E+02	10003		
7	PrimS4R1	-8.1125000000E+02	-1.1863291400E+00	-1.5220805640E+02	10004		
8	PrimS5R1	-8.1125000000E+02	-2.6099395028E+02	-3.0220805640E+02	10005		
9	PrimS6R1	-8.1125000000E+02	-2.6099395028E+02	-6.0220805640E+02	10006		
10	PrimS7R1	-8.0000000000E+02	-1.1863291420E+00	-4.5220805640E+02	10007		
11	AC Flat	-3.0046370000E+03	-1.1863291420E+00	-4.5220805640E+02	10009		
12	PrimS1R2	-8.1125000000E+02	-1.1863291400E+00	-7.5220805640E+02			
13	PrimS2R2	-8.1125000000E+02	2.5862129199E+02	-6.0220805640E+02			
14	PrimS3R2	-8.1125000000E+02	2.5862129199E+02	-3.0220805640E+02			
15	PrimS4R2	-8.1125000000E+02	-1.1863291400E+00	-1.5220805640E+02			
16	PrimS5R2	-8.1125000000E+02	-2.6099395028E+02	-3.0220805640E+02			
17	PrimS6R2	-8.1125000000E+02	-2.6099395028E+02	-6.0220805640E+02			
18	PrimS7R2	-8.0000000000E+02	-1.1863291420E+00	-4.5220805640E+02			
19	SecondaryR2	-2.4176370000E+03	-1.1863291420E+00	-4.5220805640E+02			
20	TelFoldFlatR2	0.0000000000E+00	-1.1863291420E+00	-4.5220805640E+02			
21	Beamsplitt1FT	0.0000000000E+00	-3.9610090070E-01	-1.0975096820E+02			
22	Beamsplitt1BT	0.0000000000E+00	-3.9610090070E-01	-1.0975096820E+02			
23	OAP1	0.0000000000E+00	2.2708649370E+02	2.0192710870E+03	10012		
24	Flat1	0.0000000000E+00	-1.7530089870E+02	4.7959507390E+02	10013	FEM Coordinates vs Optical Coordinates	
25	DM	0.0000000000E+00	3.0868709890E+02	2.1222981870E+03	10014		
26	OAP2	0.0000000000E+00	4.7198529030E+02	5.8904949880E+02	10015	FEM	MACOS
27	FSM	0.0000000000E+00	4.5227665110E+02	1.9410828090E+03	10016	+X	+Z
28	Beamsplitt2FR	0.0000000000E+00	5.3772423140E+02	1.6430916830E+03	10017	+Y	+Y
29	Flat2	0.0000000000E+00	7.5239919080E+02	1.6430470220E+03	10018	+Z	-X
30	CCD_Return	2.7516654780E-07	7.4059952480E+02	1.4289859400E+03			
31	Exit Pupil	-5.3467368230E-07	1.2009846570E+03	9.8373065960E+03			
32	CCD	2.7516654780E-07	7.4059952480E+02	1.4289859400E+03	10019		
shaded elements are used in linear optics model						all dimensions in mm	

Dynamic Equations Of Motion

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{k}\mathbf{u} = \mathbf{p}(t)$$

Apply the standard coordinate transformation

$$\mathbf{u}(t) = \Phi\mathbf{q}(t)$$

To obtain

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{P}(t)$$

where

$$\mathbf{M} = \Phi^T\mathbf{m}\Phi = \text{modal mass matrix} = \mathbf{I} \text{ when } \Phi's \text{ are mass normalized}$$

$$\mathbf{C} = \Phi^T\mathbf{c}\Phi = \text{modal damping matrix} = diag(2zw) \text{ when } \Phi's \text{ are mass normalized}$$

$$\mathbf{K} = \Phi^T\mathbf{k}\Phi = \text{modal stiffness matrix} = diag(w^2) \text{ when } \Phi's \text{ are mass normalized}$$

$$\mathbf{P} = \Phi^T\mathbf{p} = \text{modal force vector}$$

Alternative Formulation

Define in terms of relative motion

$$\mathbf{w} = \mathbf{u} - \mathbf{z}$$

$$\mathbf{m}\ddot{\mathbf{w}} + \mathbf{c}\dot{\mathbf{w}} + \mathbf{k}\mathbf{w} = -\mathbf{m}\ddot{\mathbf{z}}$$

Apply the standard coordinate transformation

$$\mathbf{w}(t) = \Phi\mathbf{q}(t)$$

To obtain

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = -\Phi\mathbf{m}\ddot{\mathbf{z}}$$

Assumptions for Dynamics Model

- FEM includes elastic elements representing vibration isolation; springs tuned at 1 Hz (vertical) and 1.6 Hz (horizontal) with assumed modal damping of 20%
- Assumed 1% (Q = 50) modal damping for all other modes

Reduction to State-Space Form, I

Equation of motion in generalized coordinates

$$\ddot{\mathbf{q}} + diag(2zw)\dot{\mathbf{q}} + diag(w^2)\mathbf{q} = \Phi\mathbf{p}(t)$$

Physical coordinates as function of modal coordinates

$$\mathbf{u}(t) = \Phi\mathbf{q}(t)$$

LOS error as function of physical coordinates

$$\Theta(t) = \mathbf{K}\mathbf{u}(t)$$

Define the state vector thusly...

$$\mathbf{X} = \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix}$$

Reduction to State-Space Form, II

And the exercise is left to the reader to obtain...

$$\begin{bmatrix} \dot{\mathbf{q}} \\ \ddot{\mathbf{q}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -diag(w_i^{\ 2}) & -diag(2zw_i) \end{bmatrix} \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \Phi^T \end{bmatrix} \mathbf{p}(t)$$

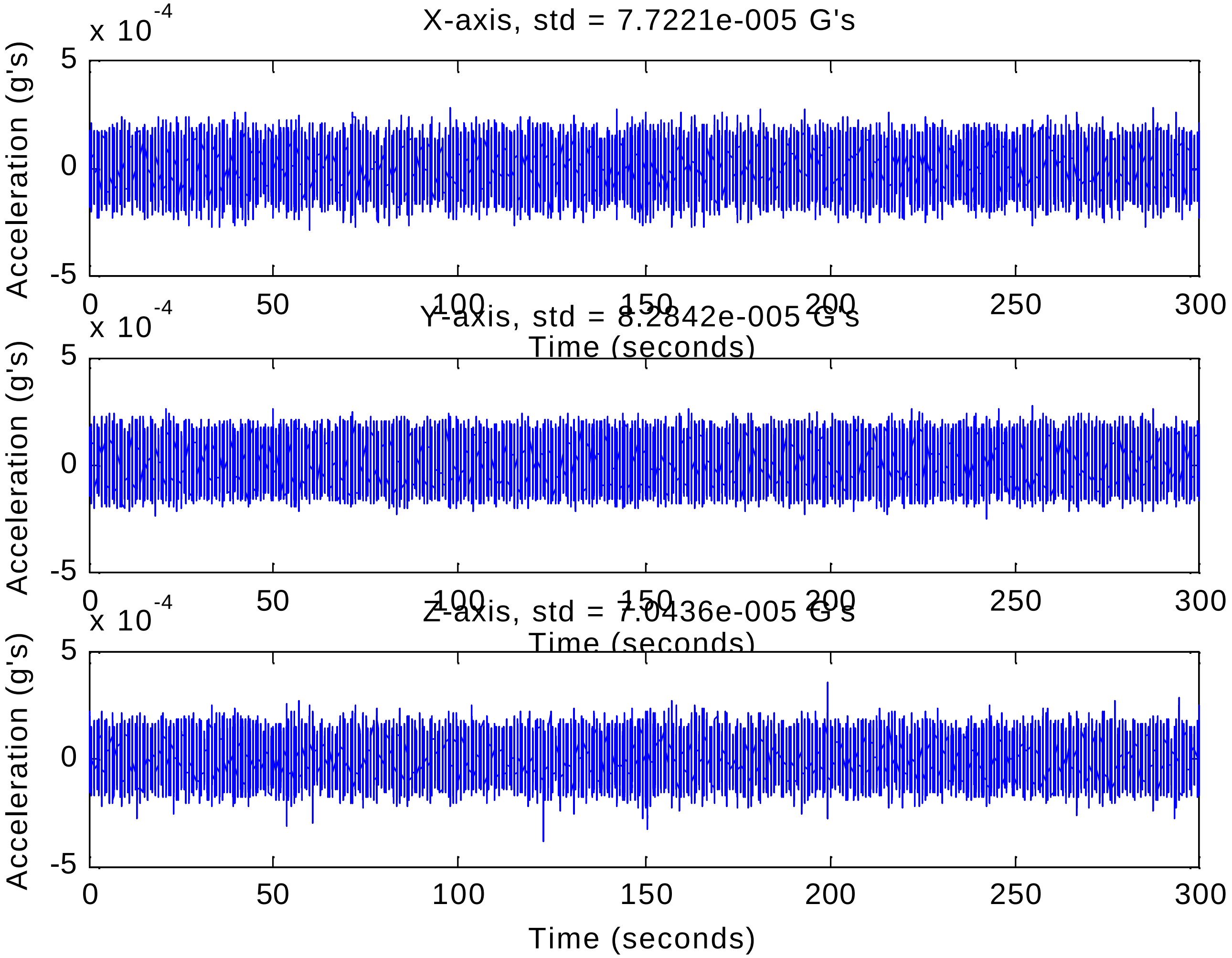
$$\Theta(t) = \mathbf{K} \begin{bmatrix} \Phi & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix}$$

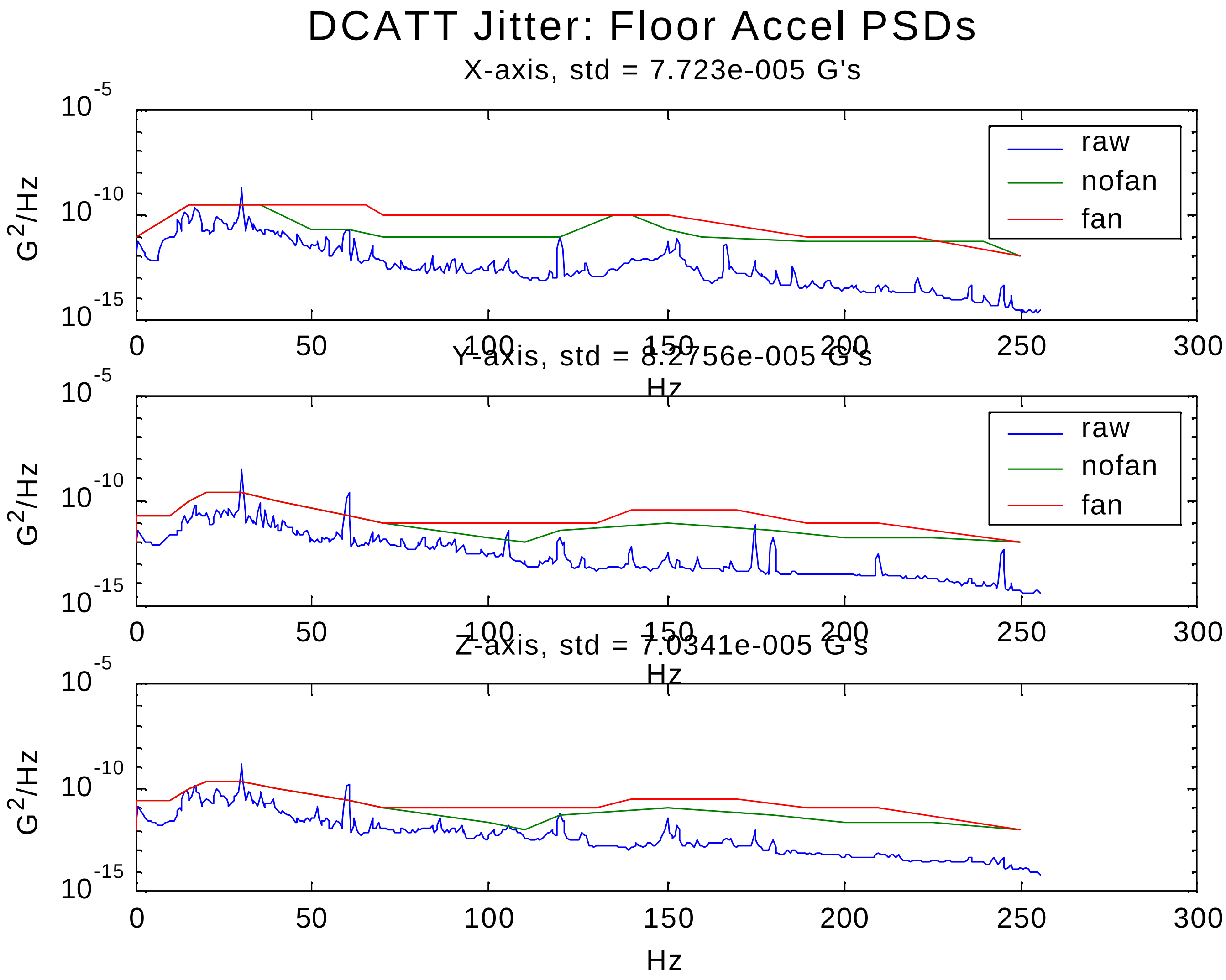
Which is a state-space system with A,B,C defined in terms of the eigenvalues, eigenvectors, and optical sensitivities

Accelerometer Data

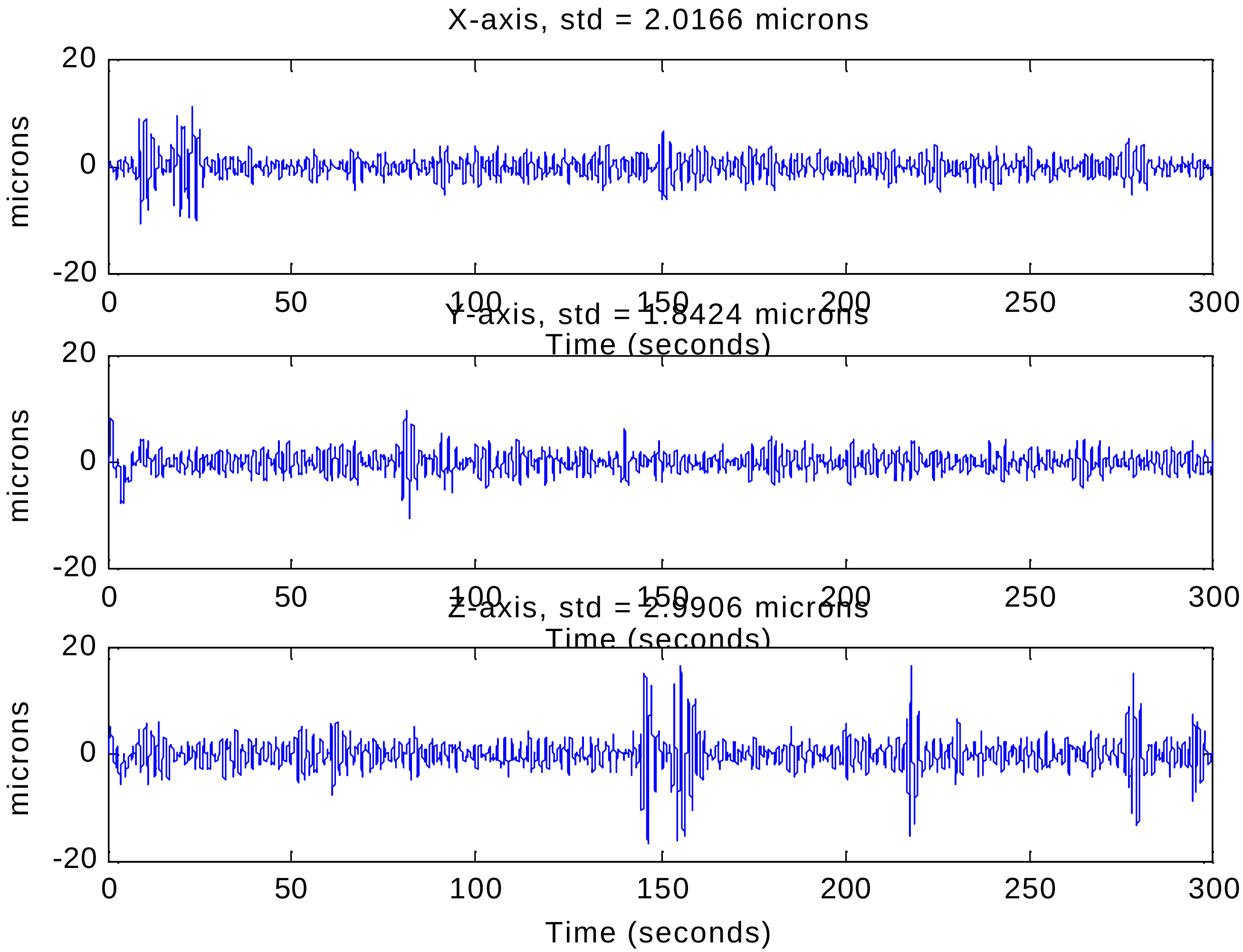
- Sensitive but possibly not lowest noise
- There are peaks at 30, 60, 120, etc. Hz that may be EM noise, but there may actually be floor motion at those frequencies due to machinery oscillating at 1800 rpm or some such speed
- Should repeat the seismic survey and take data sufficient to clarify this (e.g. suspend an accel in mid-air to determine if the signal is electromagnetic or truly mechanical motion)
- Acceleration levels for 300-second time histories are approximately 8e-5 g rms in all 3 axes, or about 2 microns rms displacement (plots on page 7 and 8)
- Applied a high-pass filter to the raw time-series data to take out drift and bias
- Plots of accel time and PSD data, and displacement time and PSD data, follow
- Envelopes (conservative) used by Sandra/Jonathan for independent NASTRAN-based analysis are shown on accel PSD plots (page 9), and drive the structure with about 15e-4 g's rms, or twice the rms level of raw data

DCATT Jitter: Floor Accel Time Histories




DCATT Jitter: Floor Accel PSDs
X-axis, std = 7.723e-005 G's
Y-axis, std = 8.2756e-005 G's
Z-axis, std = 7.0341e-005 G's
raw
nofan
fan
G^2/Hz
Hz

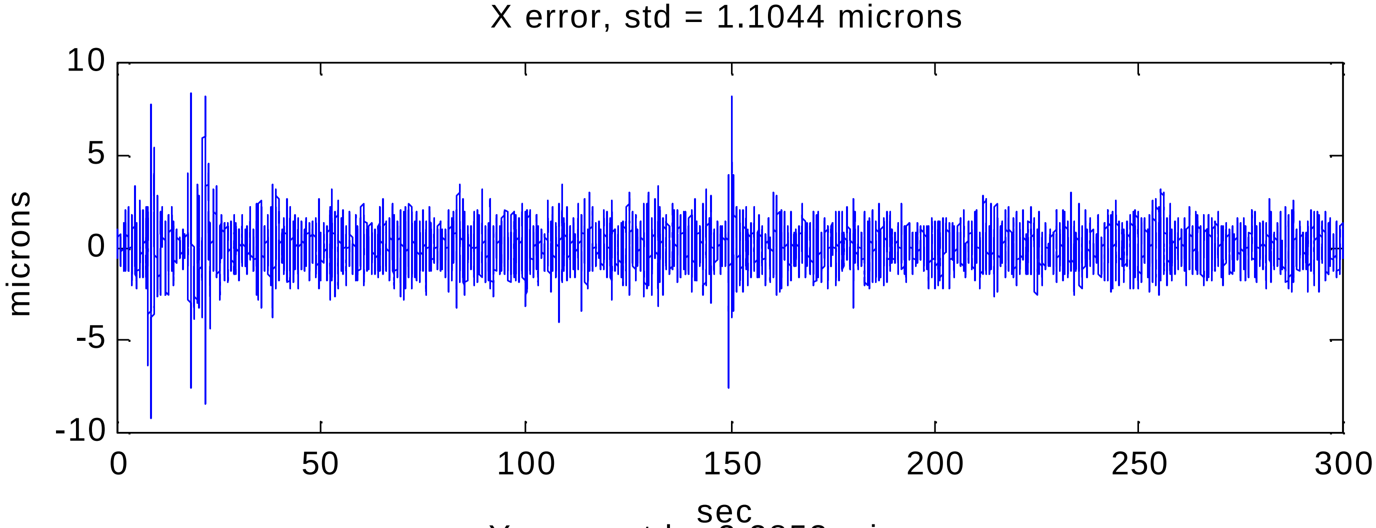
DCATT Jitter: Floor Disp Time Histories

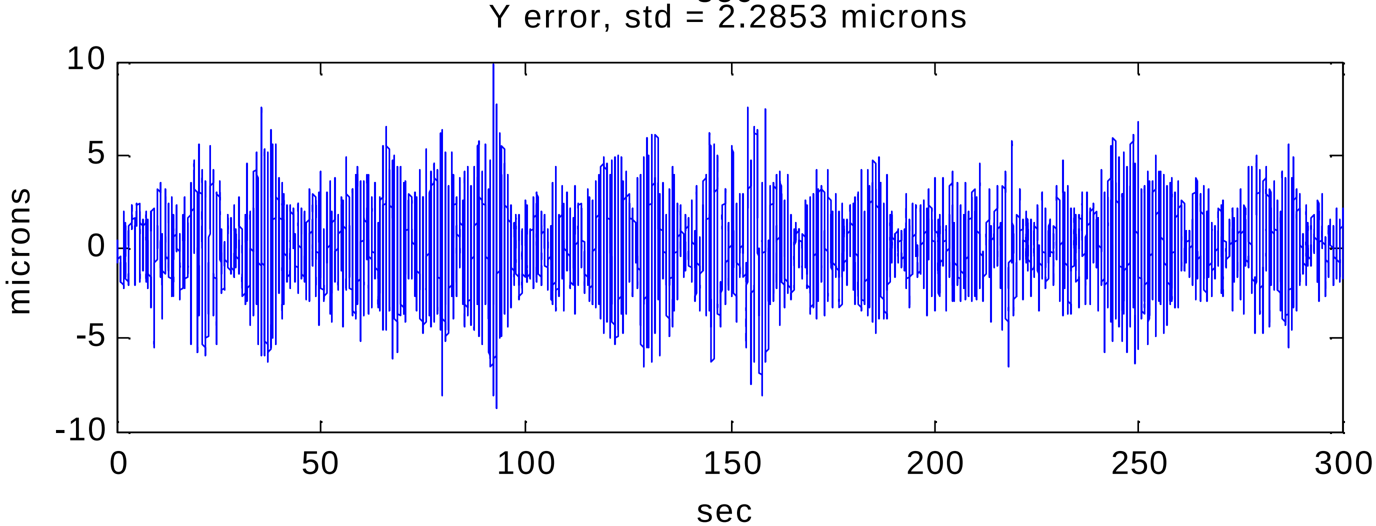


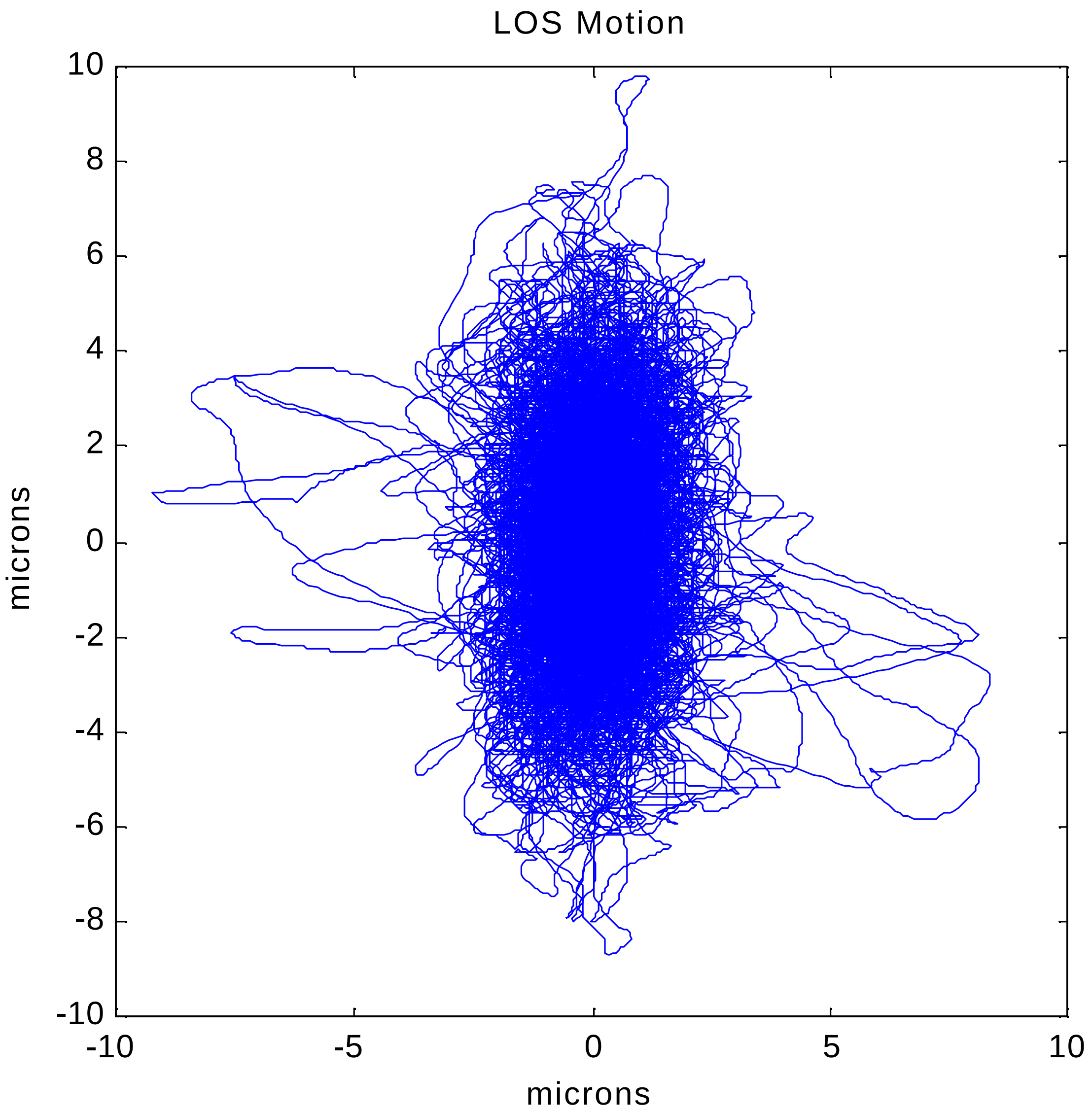
Results from Open Loop Analysis

- The results obtained via time-domain analysis show 1-σ LOS errors of 1.1 microns X-axis and 2.3 microns Y-axis, or 2.55 microns RSS'ed (plots on pages 32-33)
- The PSD data obtained by frequency-domain analysis (plots on pages 35-36) shows that the significant energy in the jitter extends out to roughly 8 Hz
- Measurements taken with bench optics only show 2-3 (1-σ) microns LOS error, well above what model would predict
  - "lab seeing" (air turbulence)
  - unmodeled disturbances (e.g. acoustics, CCD coolant loop)
  - mounts for bench optics (e.g. CCD) not entirely rigid

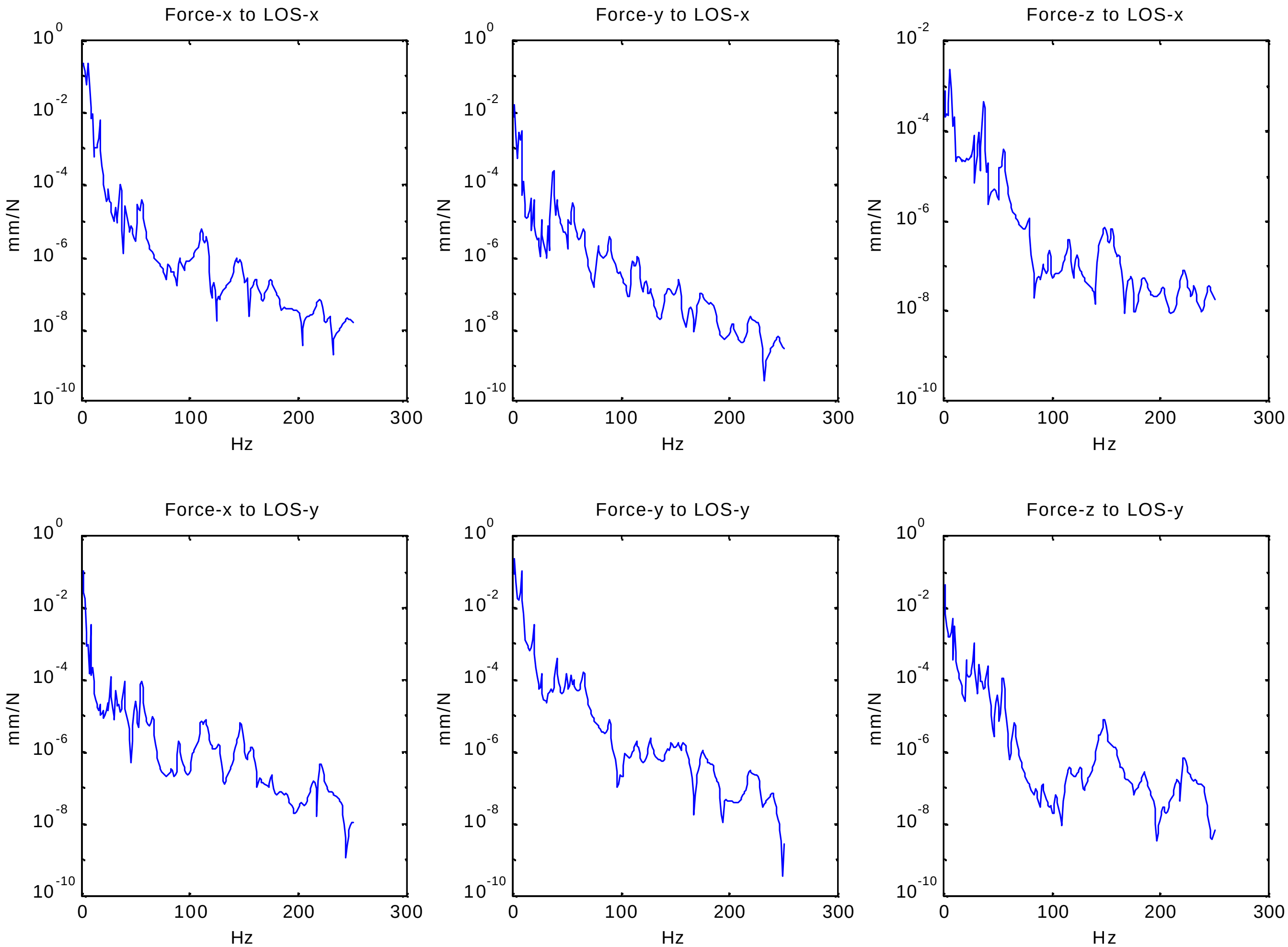
LOS Error Time Histories



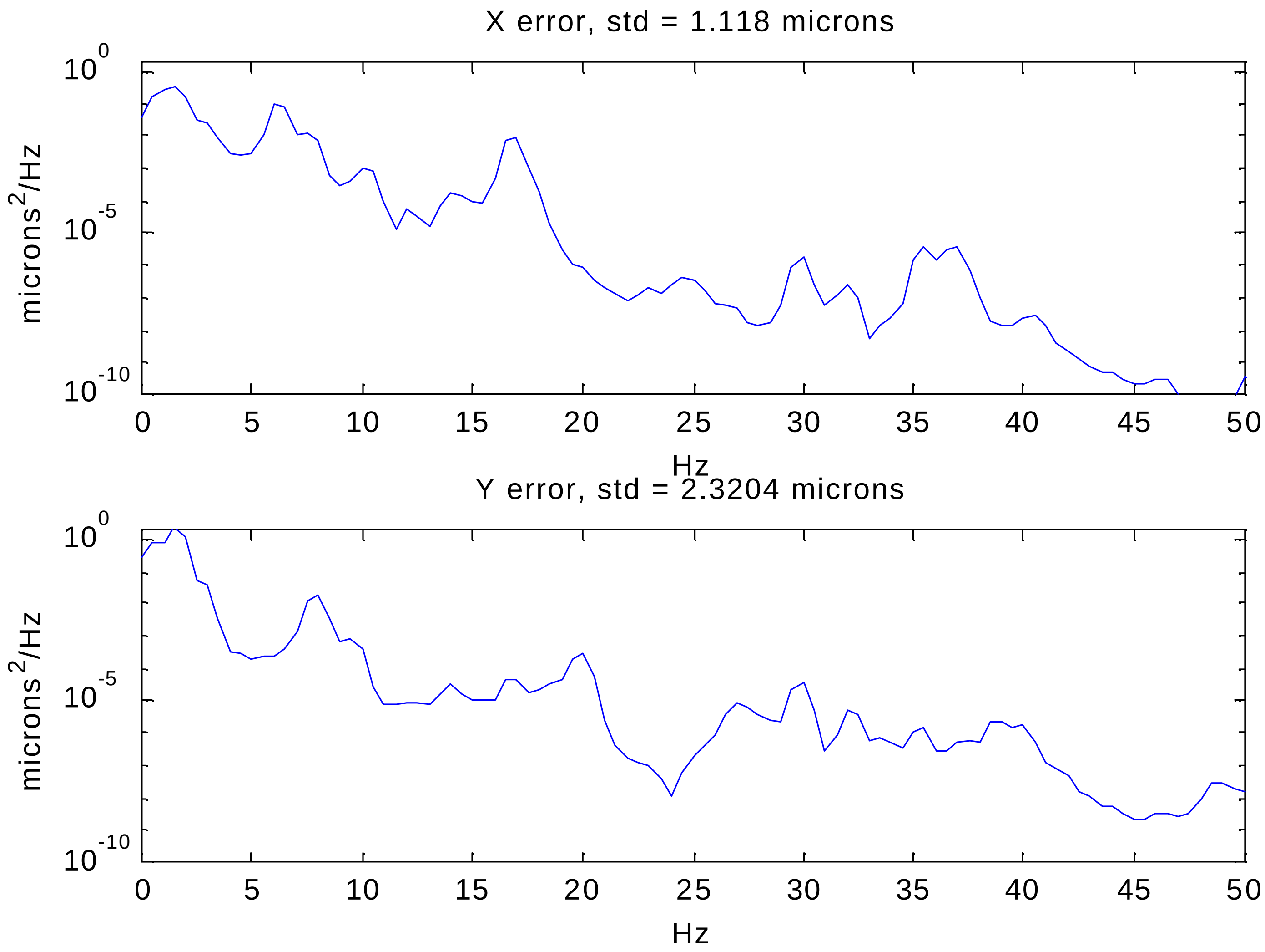



LOS Motion
10
8
6
4
2
0
-2
-4
-6
-8
-10
microns
-10
-5
0
5
10
microns

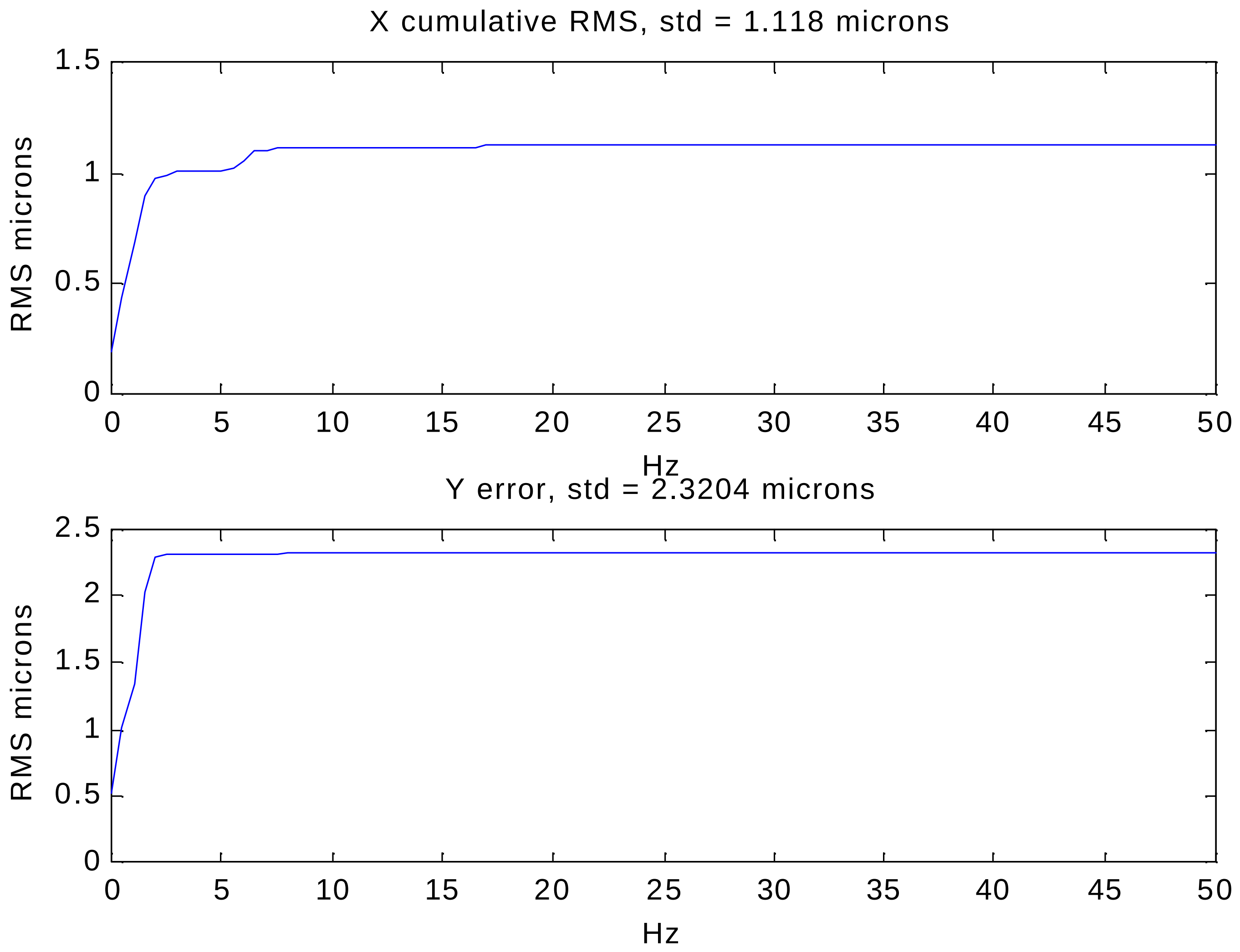
Transfer Functions - Force to LOS



Jitter PSD's

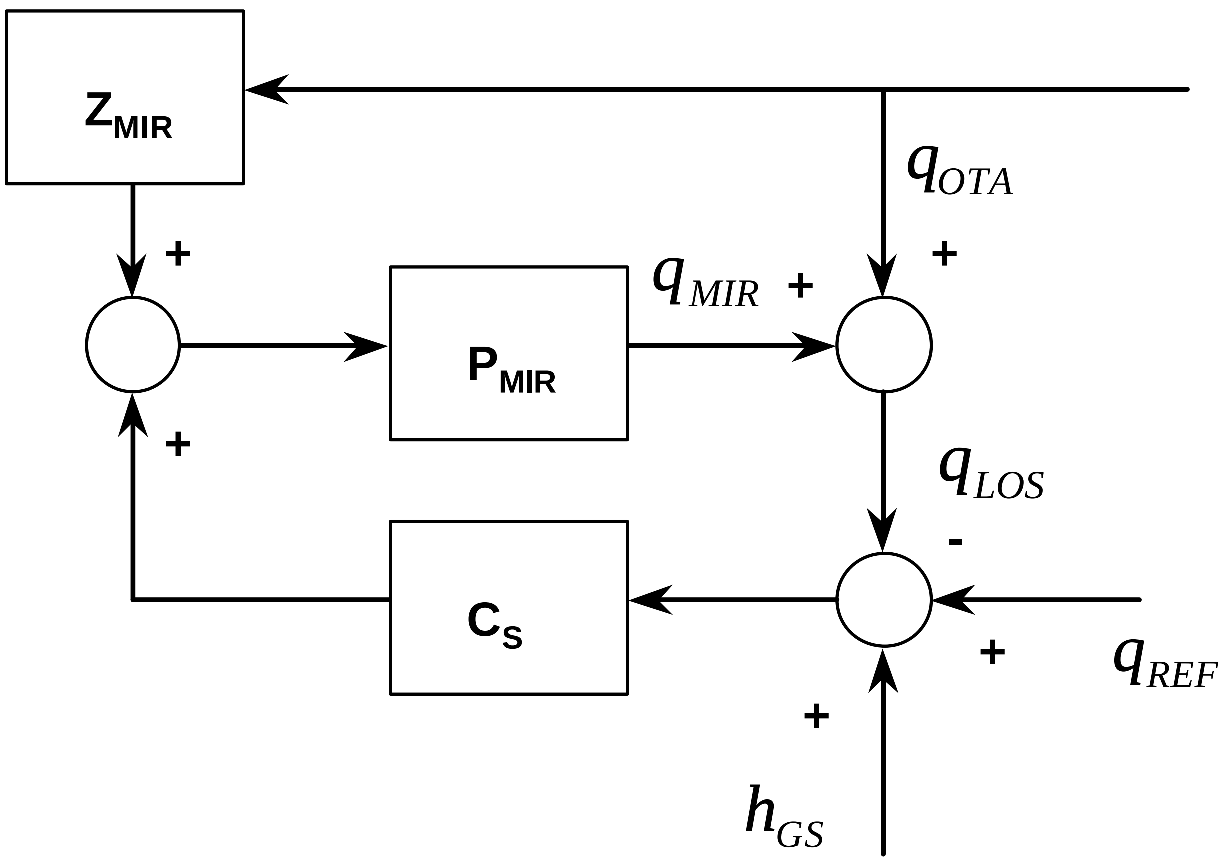

X error, std = 1.118 microns
microns^2/Hz
10^{0}
10^{-5}
10^{-10}
0 5 10 15 20 25 30 35 40 45 50
Hz
Y error, std = 2.3204 microns
microns^2/Hz
10^{0}
10^{-5}
10^{-10}
0 5 10 15 20 25 30 35 40 45 50
Hz

Jitter PSD's

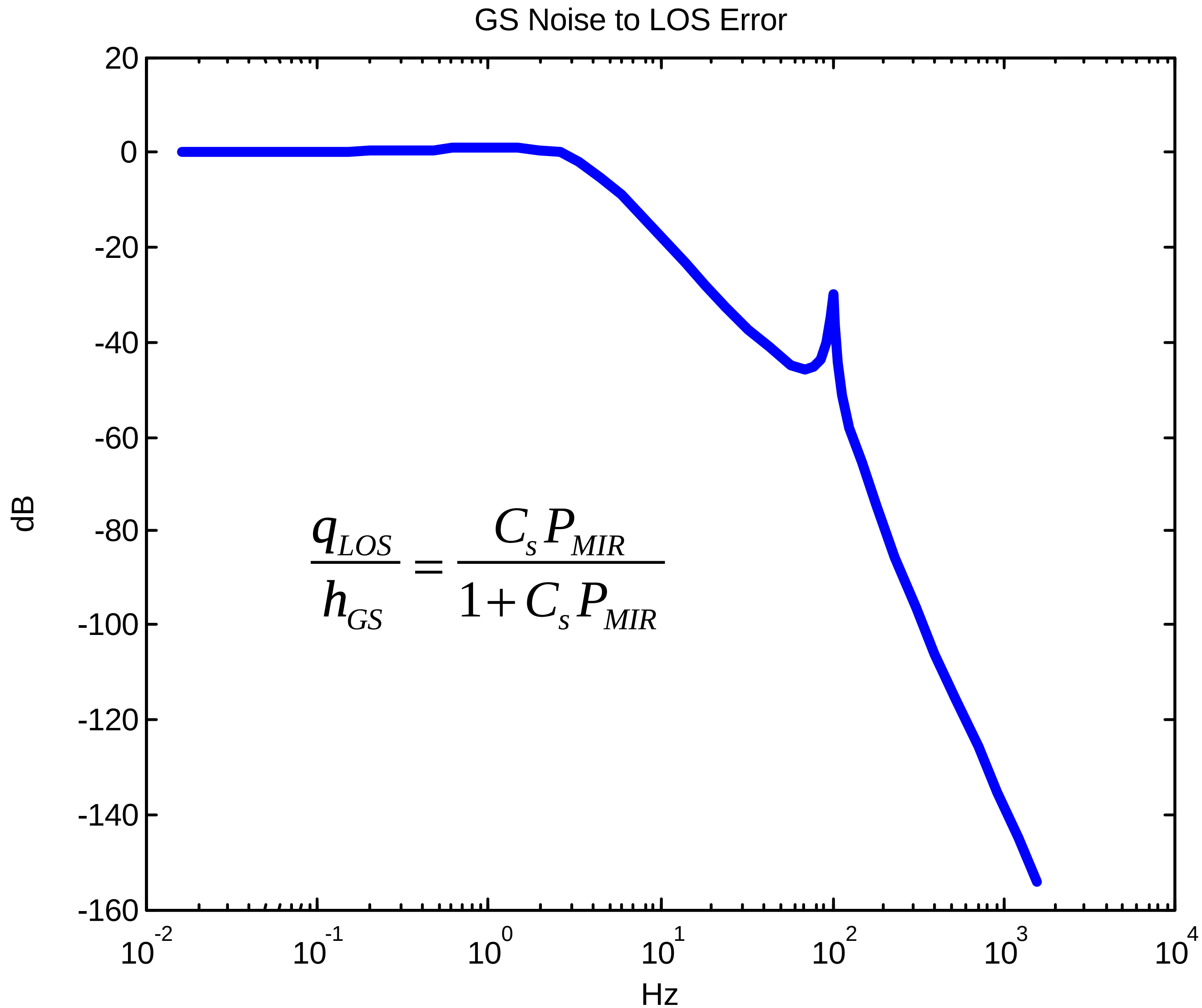

X cumulative RMS, std = 1.118 microns
RMS microns
1.5
1
0.5
0
0 5 10 15 20 25 30 35 40 45 50
Hz
Y error, std = 2.3204 microns
RMS microns
2.5
2
1.5
1
0.5
0
0 5 10 15 20 25 30 35 40 45 50
Hz

FSM Model

- Acts as a high-pass filter with respect to base motion (tip/tilt error)
- Must sense LOS error using CCD or other optical sensor (quad cell)
- Acts as low-pass filter with respect to sensor noise
- A simple single-axis model would look like

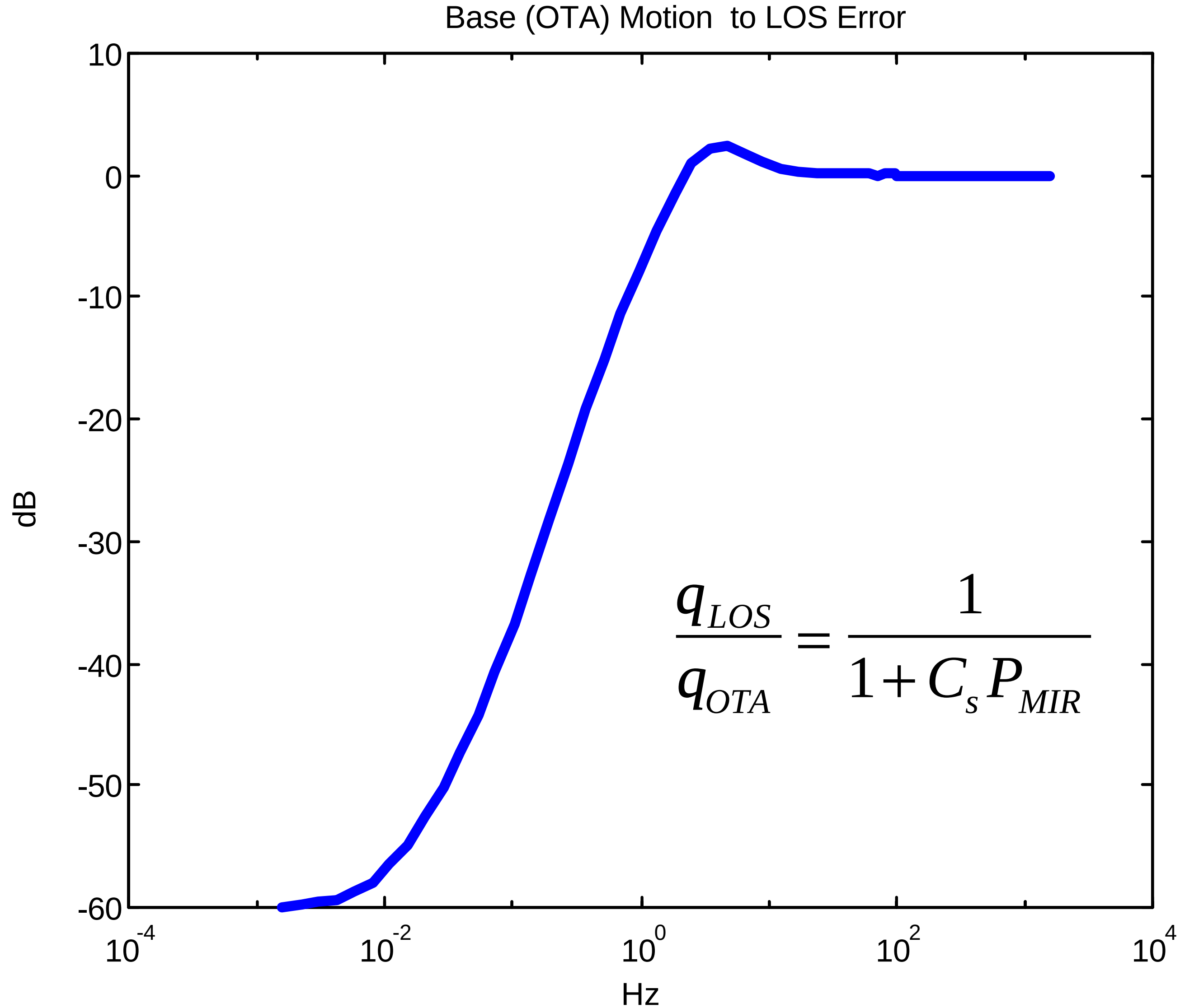


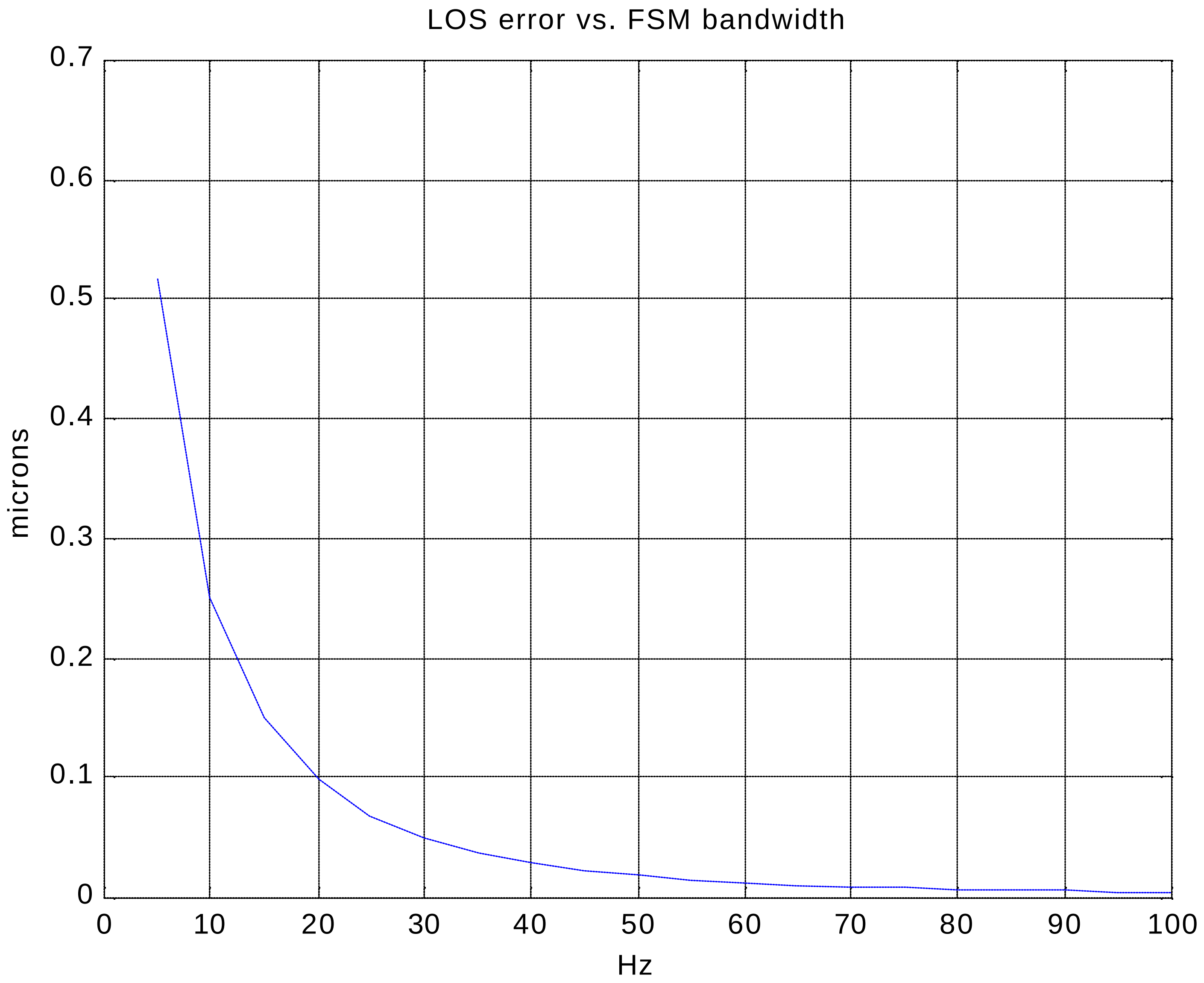
Quad Cell Sensor Noise Attenuation



$$\frac{q_{LOS}}{h_{GS}} = \frac{C_s P_{MIR}}{1 + C_s P_{MIR}}$$

FSM LOS Error Attenuation





LOS error vs. FSM bandwidth
microns
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
10
20
30
40
50
60
70
80
90
100
Hz

FSM Requirements

- The jitter requirement is 0.9 microns: required attenuation is 9dB
- Suggest a requirement of at least 20dB suppression by the FSM loop to provide margin for servo sensor noise and other unmodeled/mismodeled error sources
- A 2nd-order servo (40 dB/decade) requires 25 Hz bandwidth (250 Hz sampling) to provide adequate suppression and margin; must check radiometrics and resulting sensor noise from quad cell (see plot on page 42)